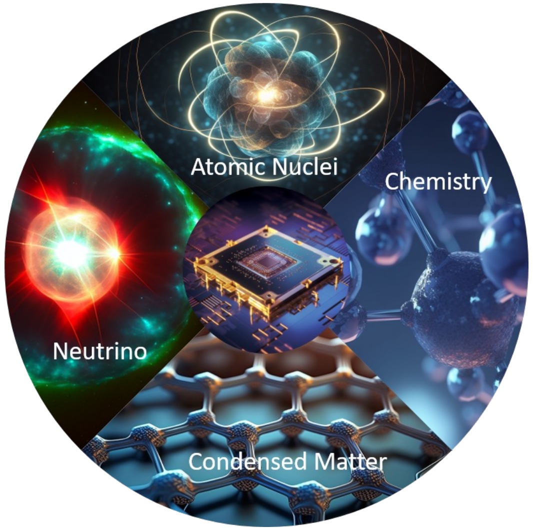
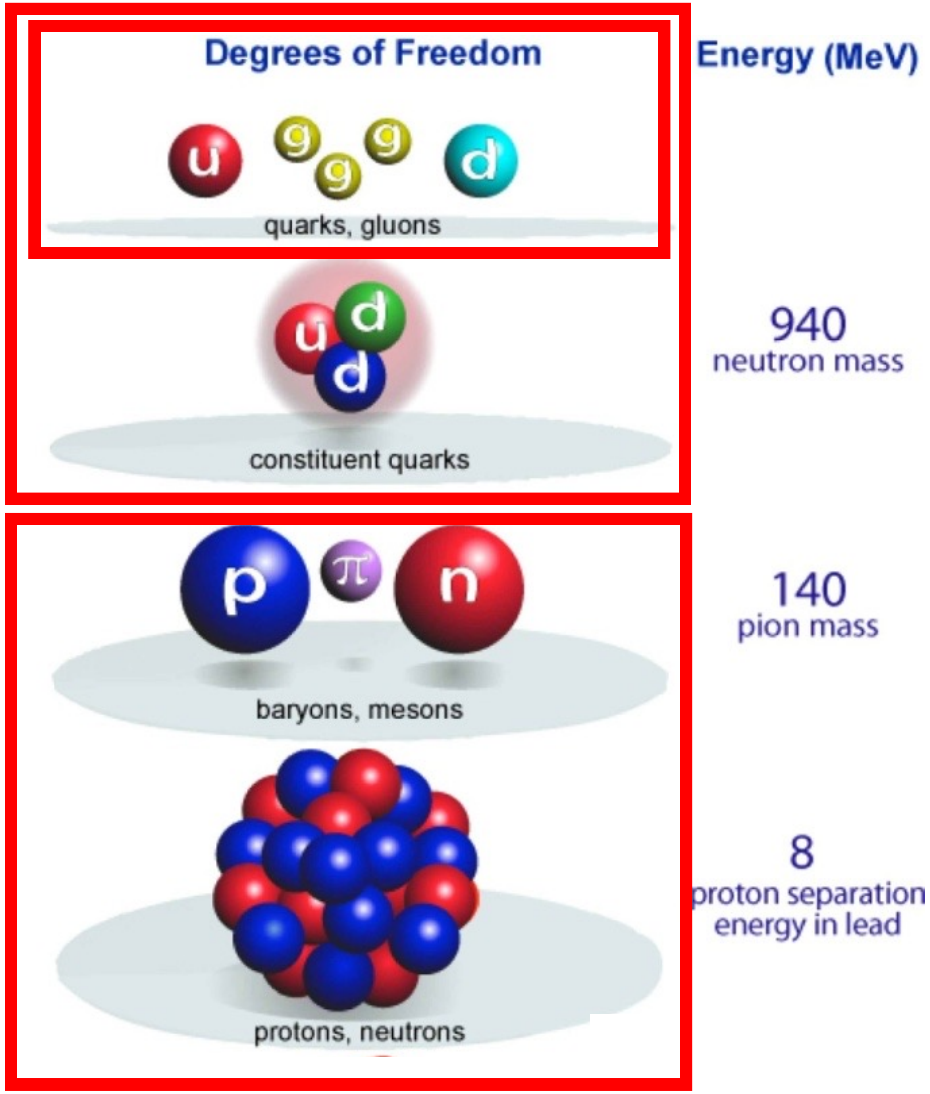
Quantum computing description of strongly interacting atomic nuclei *and neutrinos*: challenges and opportunities

Denis Lacroix

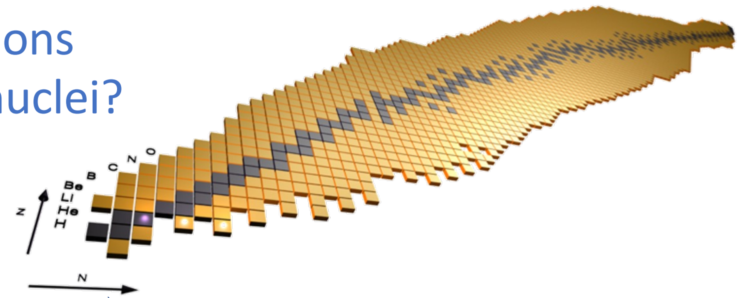


Many-body physics and QC - T. Ayral, P. Besserve, D. Lacroix, and E.A. Ruiz Guzman ,
Quantum computing with and for many-body physics, EPJA 59 (2023)
Symmetry and QC – D. Lacroix, A. Ruiz Guzman and P. Siwach,
Symmetry breaking/symmetry preserving circuits and symmetry restoration on quantum computers EPJA 59 (2023)
CERN Quantum Initiative – Di Meglio et al., Quantum Computing for High-Energy Physics: State of the Art and Challenges, PRX Quantum 5, 037001 (2024)

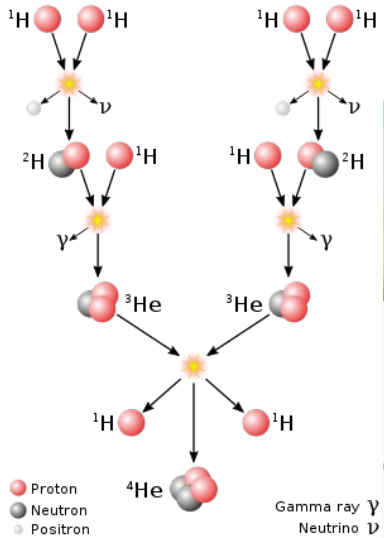
Short highlights of our fundamental science motivations

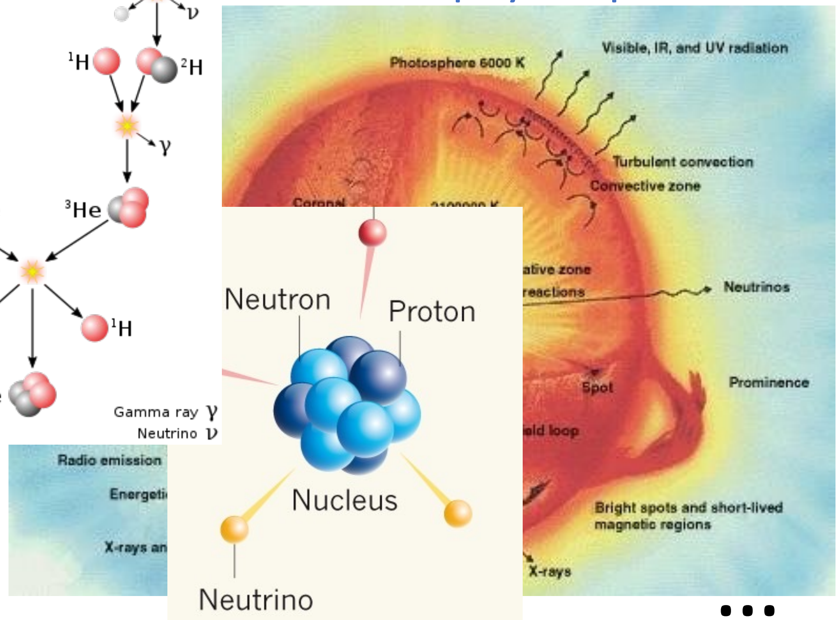


- Physics beyond the standard model
  From quarks to nucleons?
- From nucleons
  to atomic nuclei?



- QC for simulation of specific
  Astrophysics process





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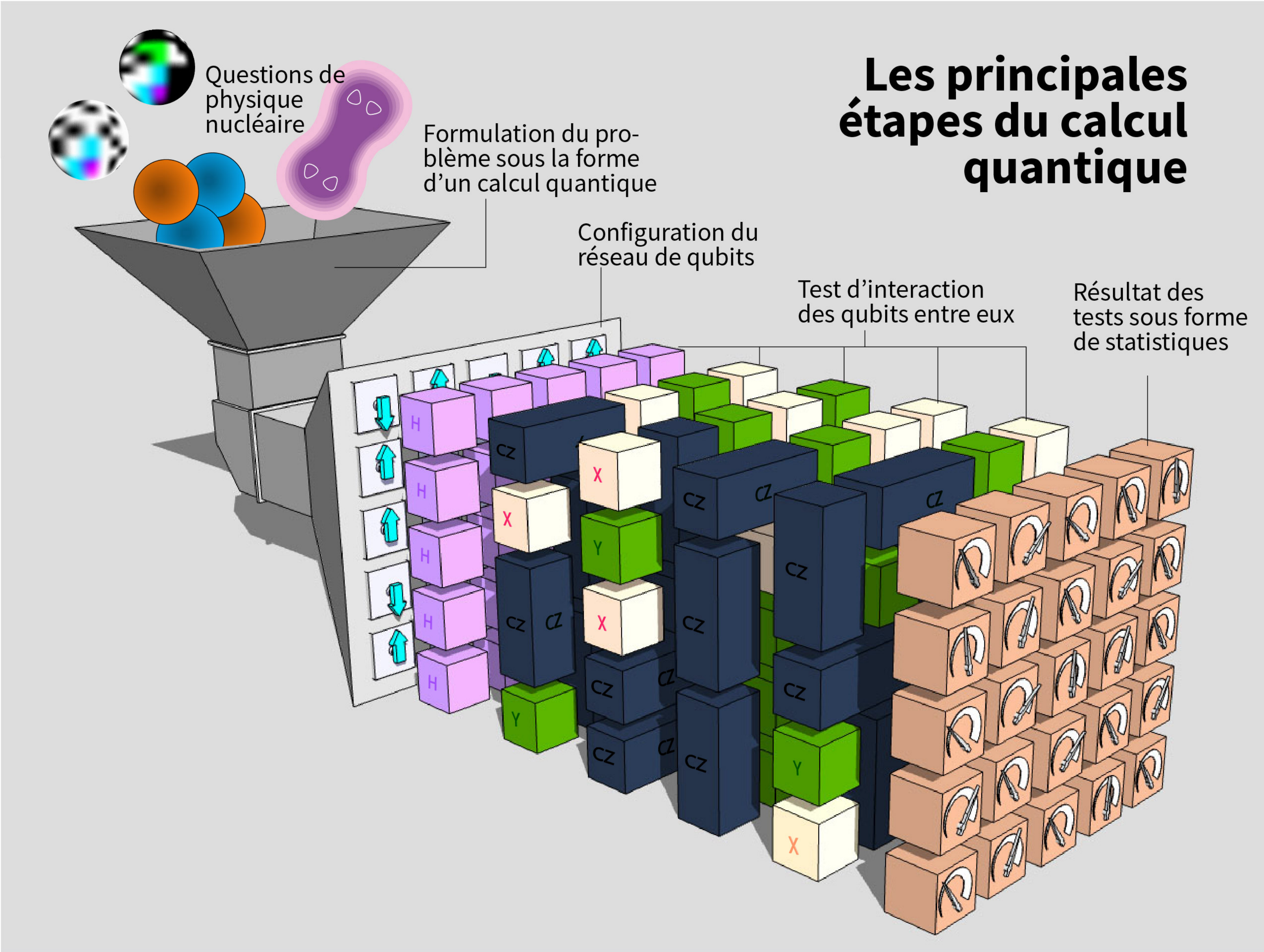
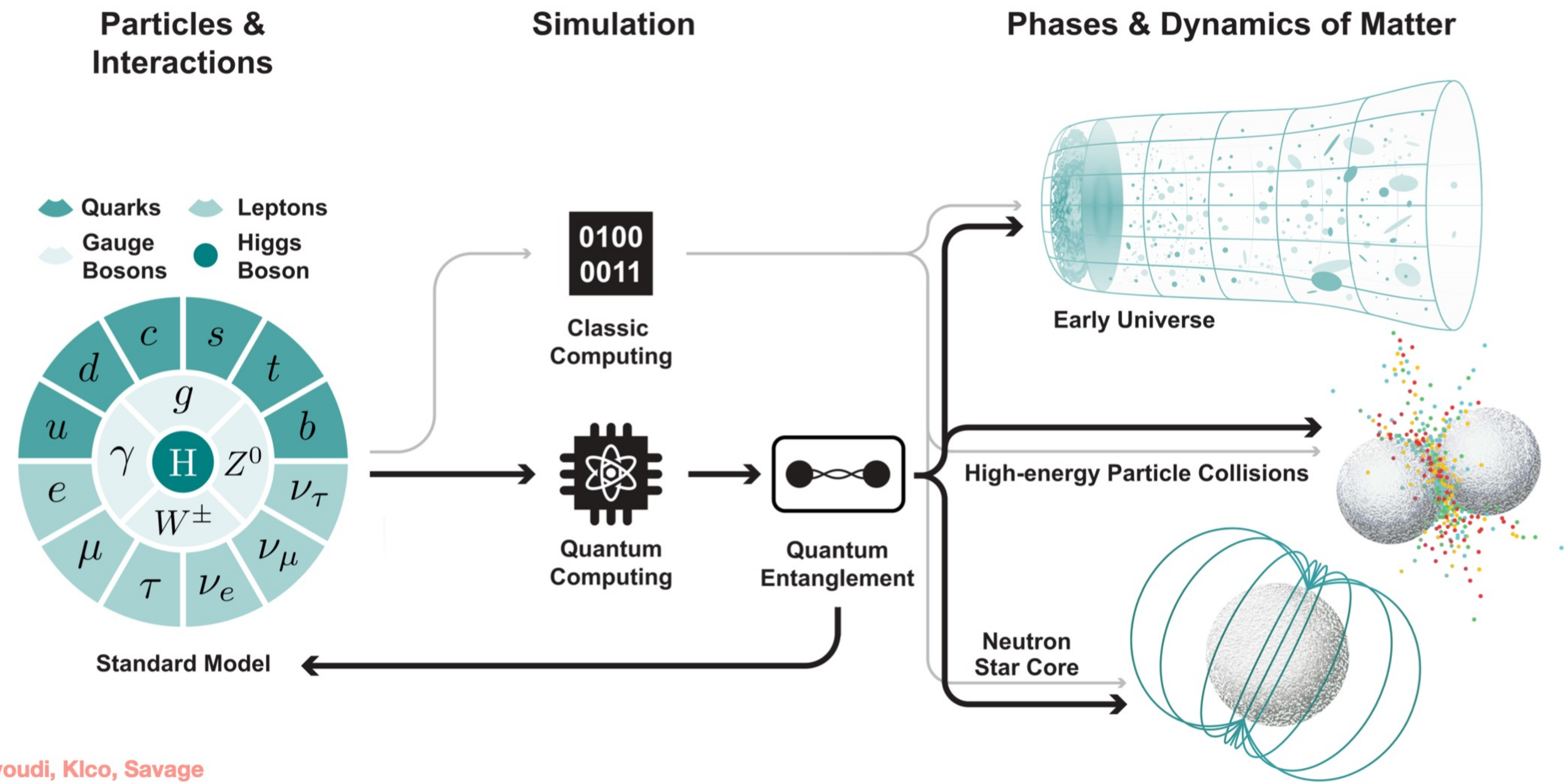


IN2P3 communication newsletter

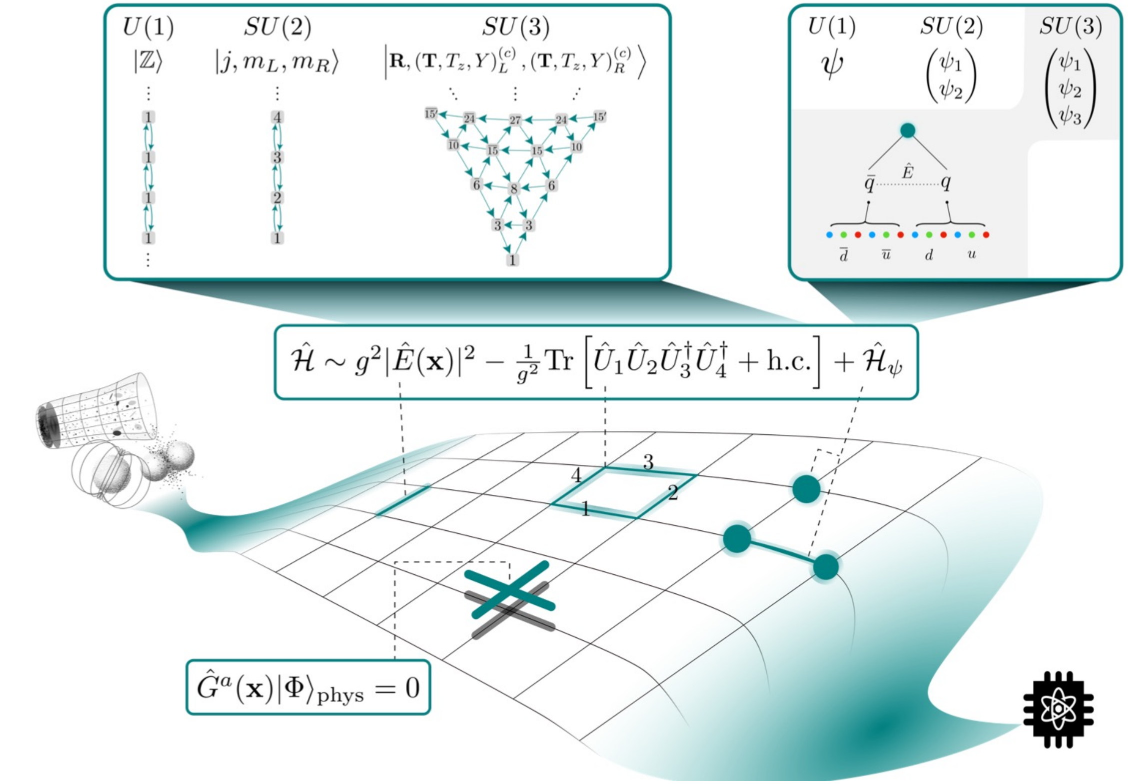
Quarks and Gluons – Effective Field theories

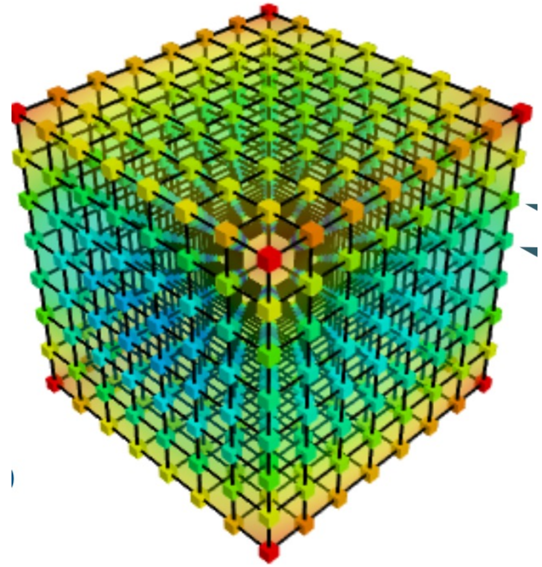


Bauer, Davoudi, Klco, Savage

Courtesy M. Savage

Digital Quantum Chromodynamics



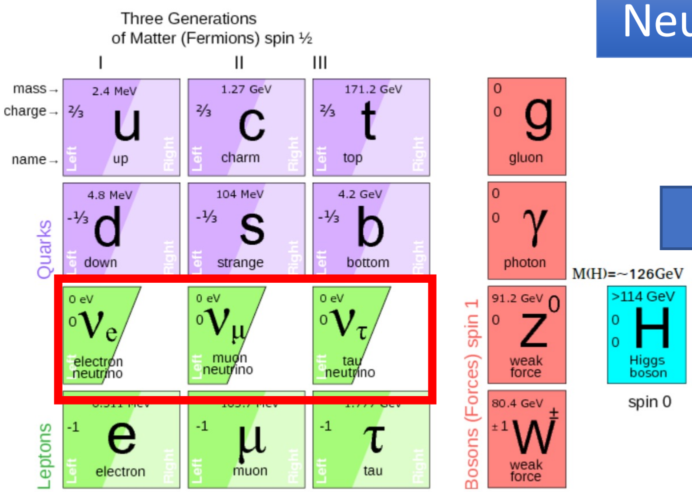


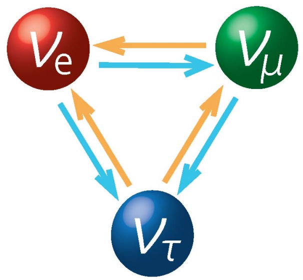
- Map quarks and gluons on quantum register
- Develop unitary operators for their evolution
- Obtain relevant observables from measurements

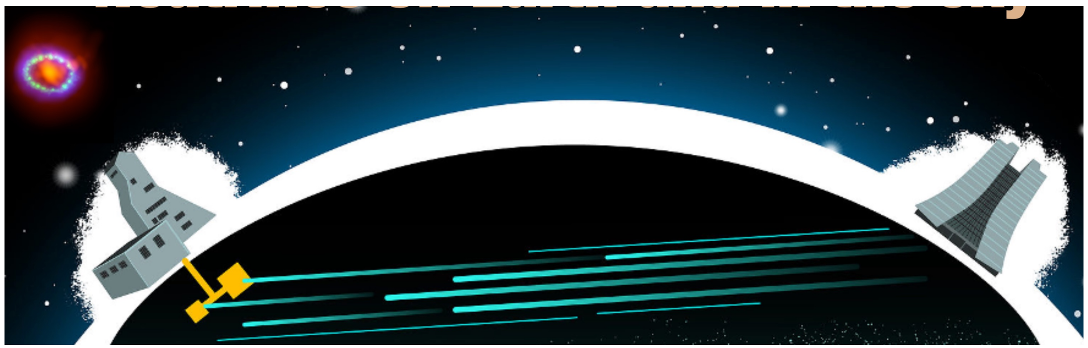
CERN Quantum Initiative – Di Meglio et al., Quantum Computing for High-Energy Physics: State of the Art and Challenges, PRX Quantum (2024)

A focus on neutrino oscillation physics simulated on quantum computers

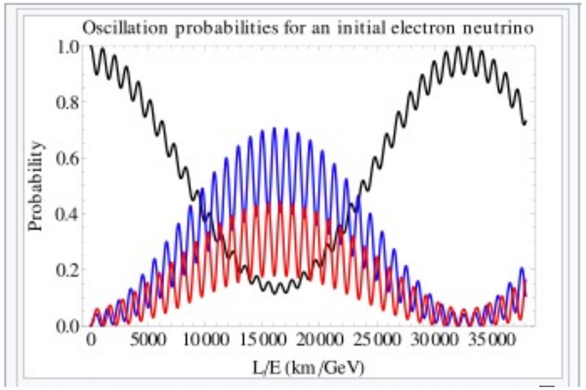
Neutrino mass and oscillations

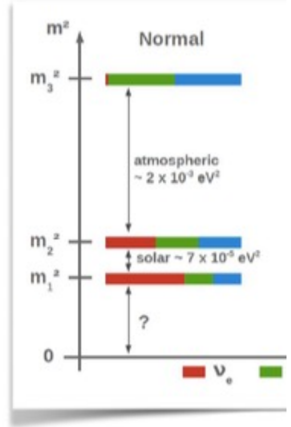






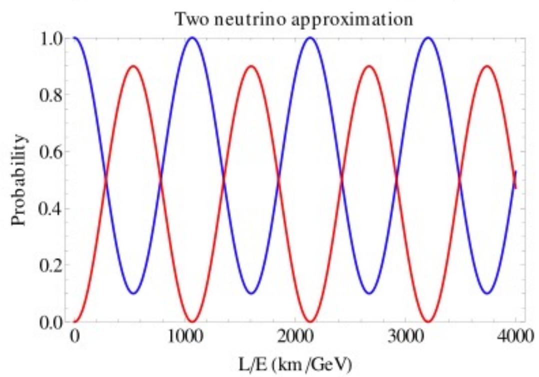
3 masses case

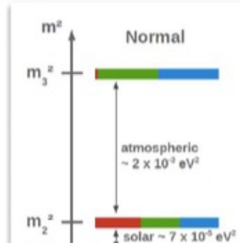




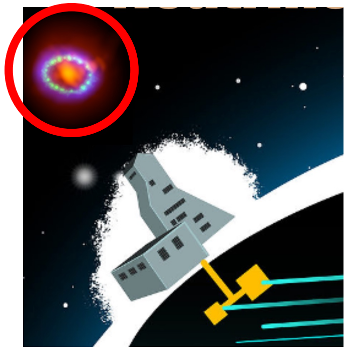
Neutrino are "natural" qutrits

2 masses approximation

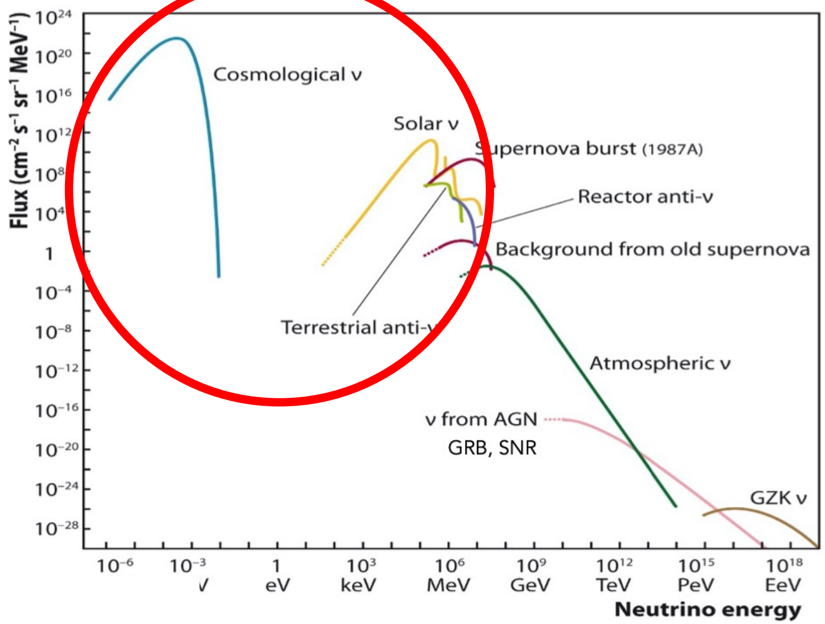




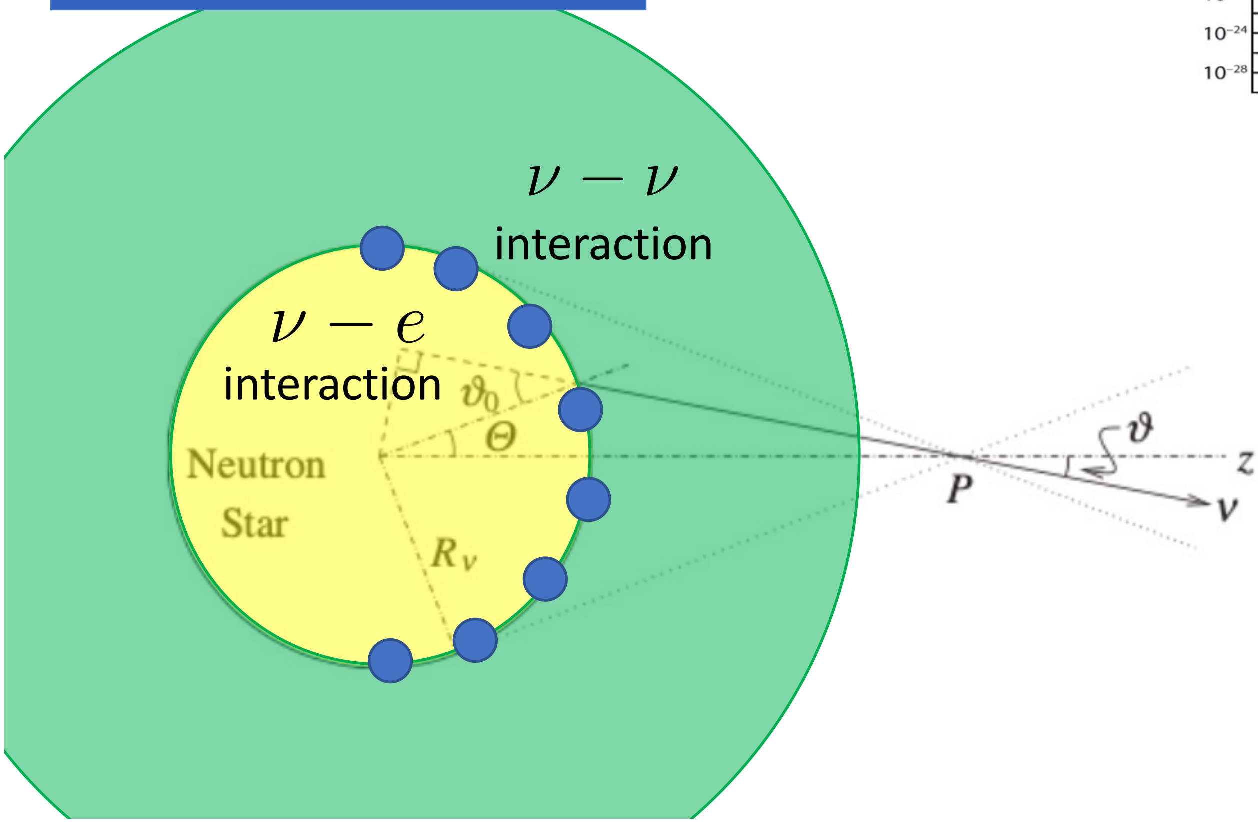
Neutrino on qubits or as qubits



Neutrino fluxes at Earth



Where is the complexity?



The problem is mapped to a many-body open quantum system problem equivalent to interacting qubits or qutrits.

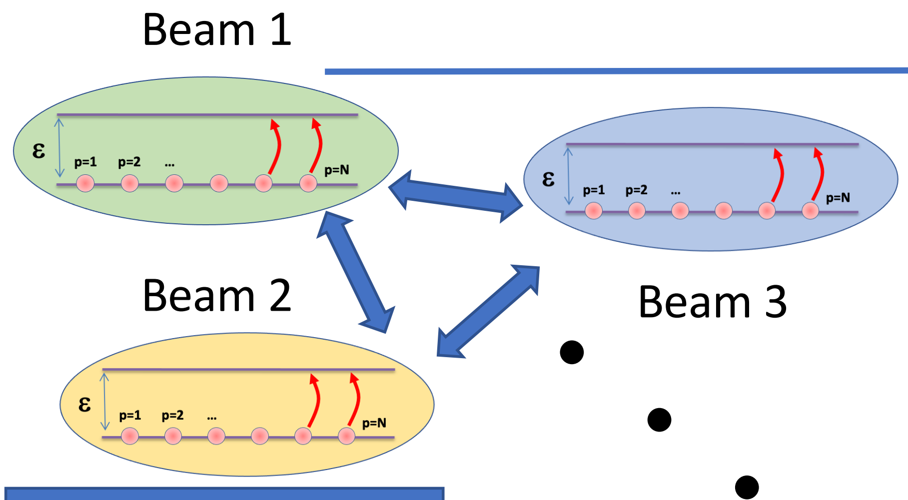


Illustration of the Hamiltonian (2 flavor approx)

Oscillation $$H_\nu = \frac{1}{N}\sum_{i=0}^{N-1} \sin(2\theta_\nu) X_i - \cos(2\theta_\nu) Z_i$$

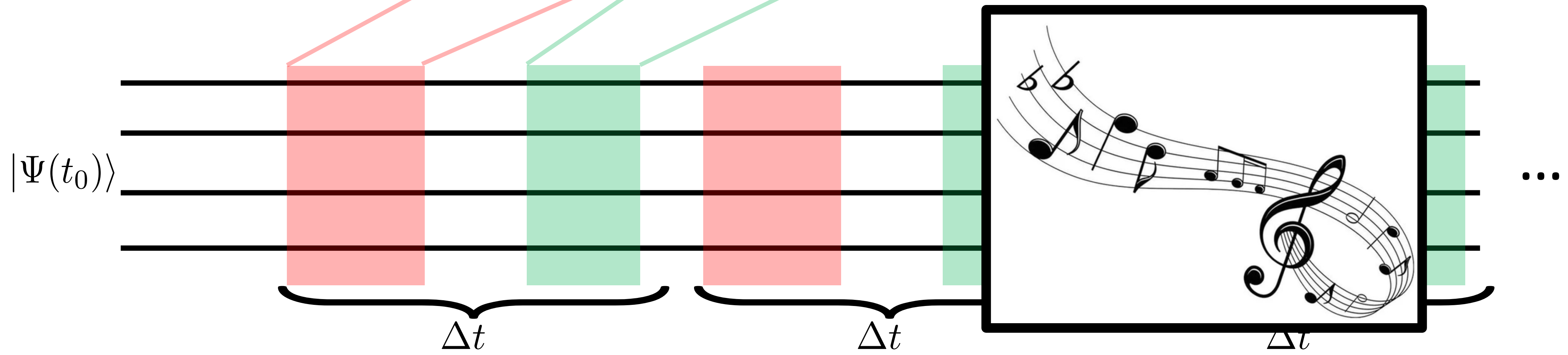
Coupling $$H_{\nu\nu} = \sum_{i<j}^{N-1} G_{i,j}[X_i X_j + Y_i Y_j + Z_i Z_j].$$

Trotterization

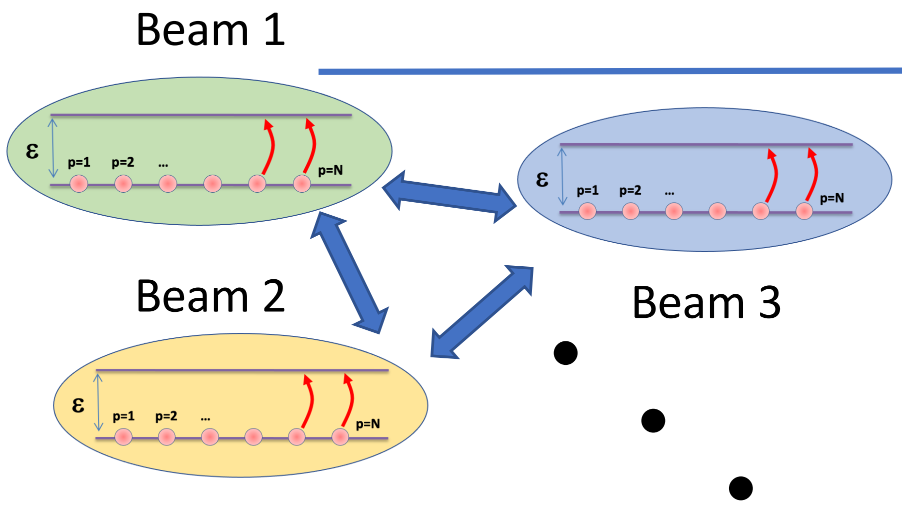
1. Decomposition of *H* into elementary blocks
2. Use a transformation (Trotter-Suzuki)

Example : $e^{i\Delta t H_1/\hbar} = e^{-i\Delta t H_1/\hbar} e^{-i\Delta t H_2/\hbar}$

3. Transforms to circuit



A focus on neutrino oscillation physics simulated on quantum computers



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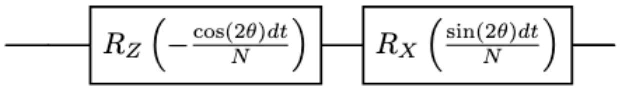

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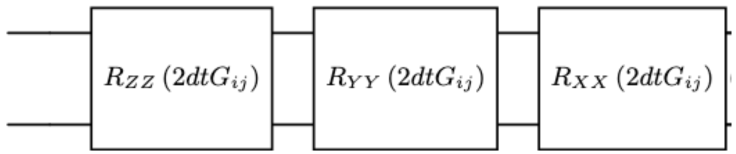
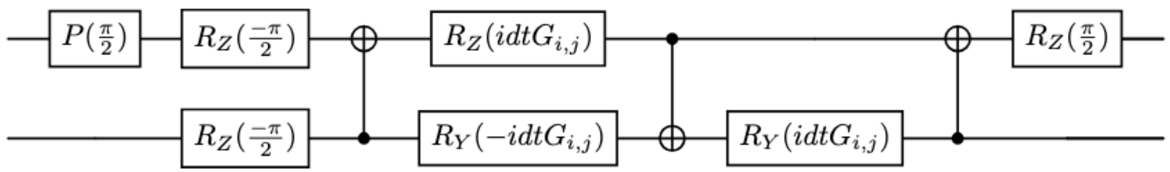


or with optimization



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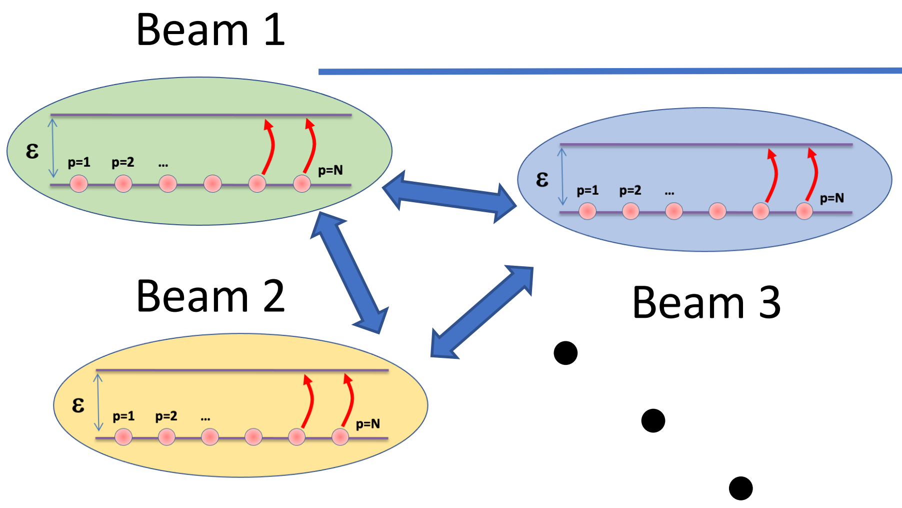

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Coupling
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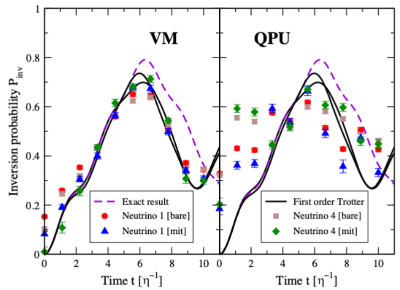
4 neutrinos
IBM-Vigo QPU

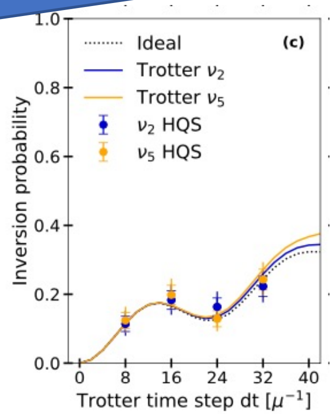
4 & 8 neutrinos
HQD-H1
Trapped Ion device

12 neutrinos
Quantinuum's H1-1
20 qubit trapped-ion

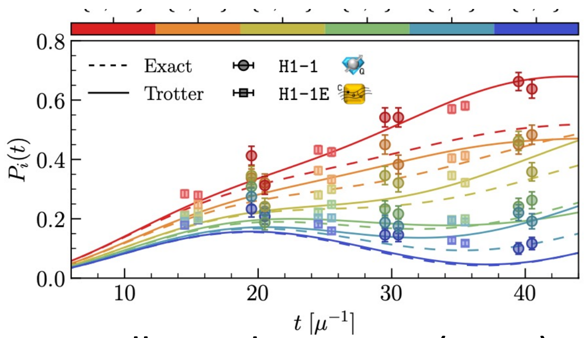
12 neutrinos / qutrits
H1-1 & ibm_torino



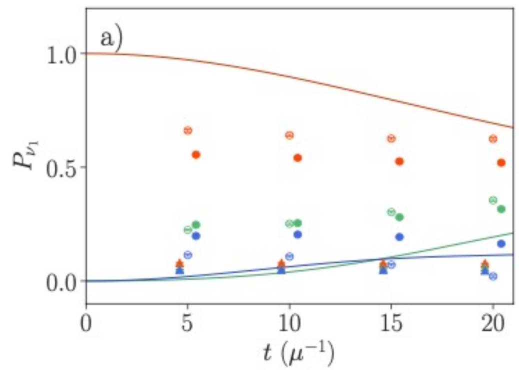
Hall et al, PRD 104 (2021)



Amitrano,et al, PRD 107, (2023)



Illa et al, PRL 130 (2023)

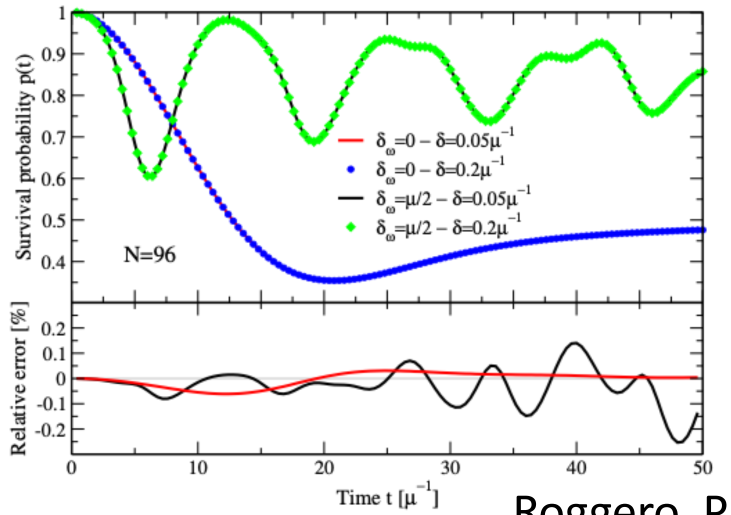


Turro et al,
arxiv:2407.13914

A focus on neutrino oscillation physics
Is also pushing the limit of classical simulation

Tensor network

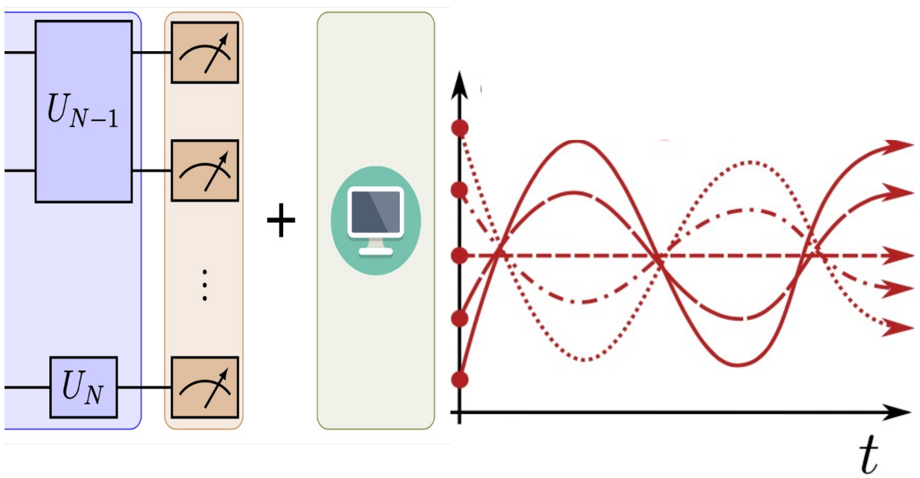
Using MPS layers to simulate neutrino evolution



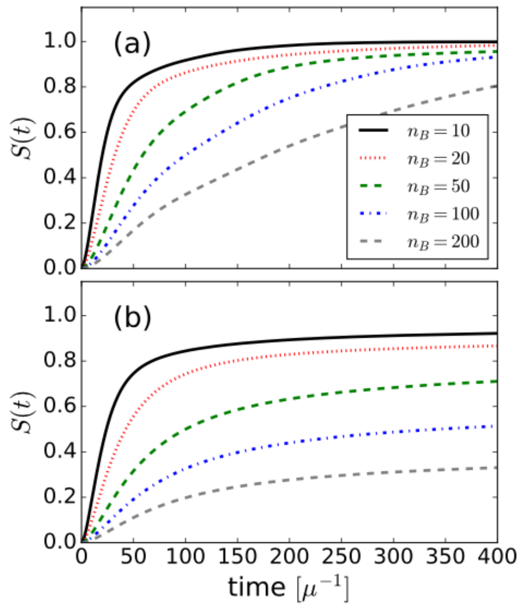
Up to ~100 neutrinos

Roggero, Phys. Rev. D 104 (2021)
Cervia et al, Phys. Rev. D 105 (2022)

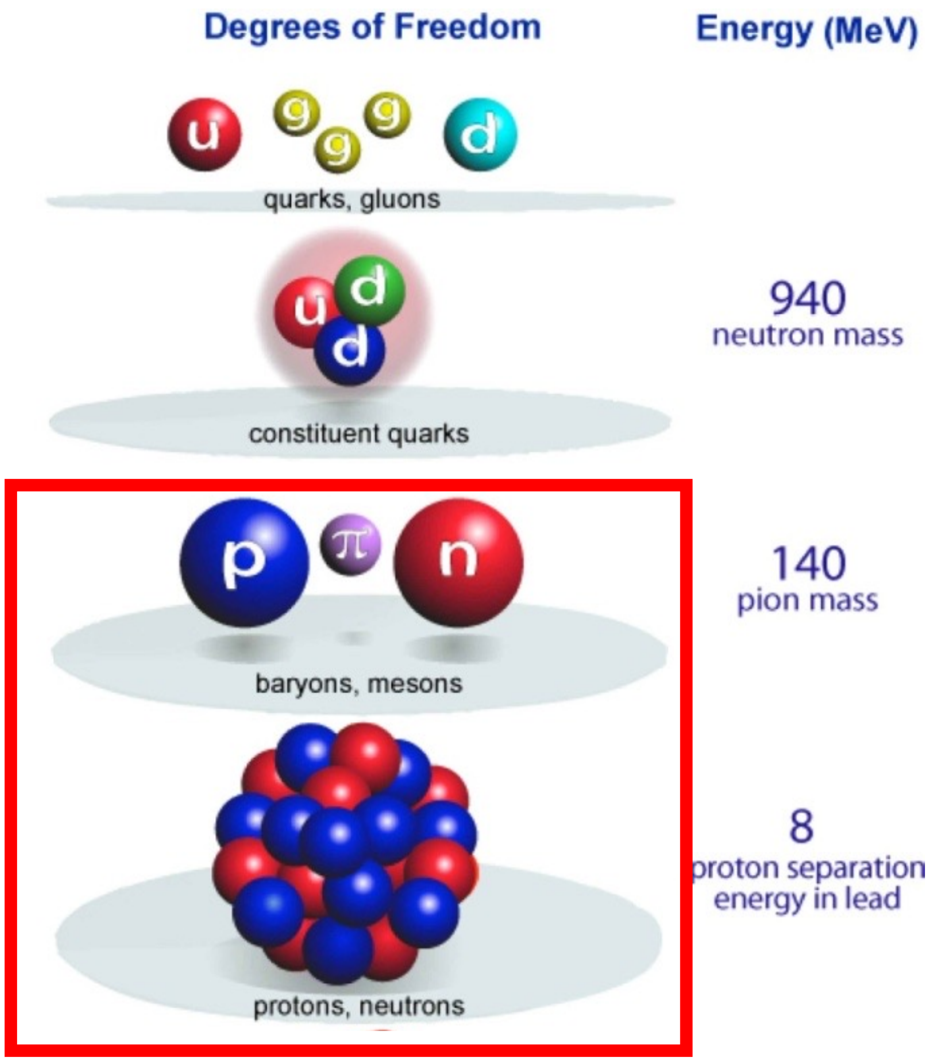
Phase-space methods



Lacroix et al, 2409.20215, PRD *in press*



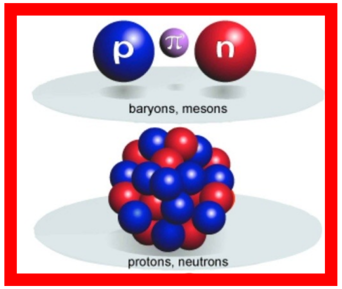
Several hundreds of neutrinos

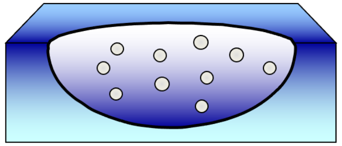


From nucleons
to atomic nuclei?

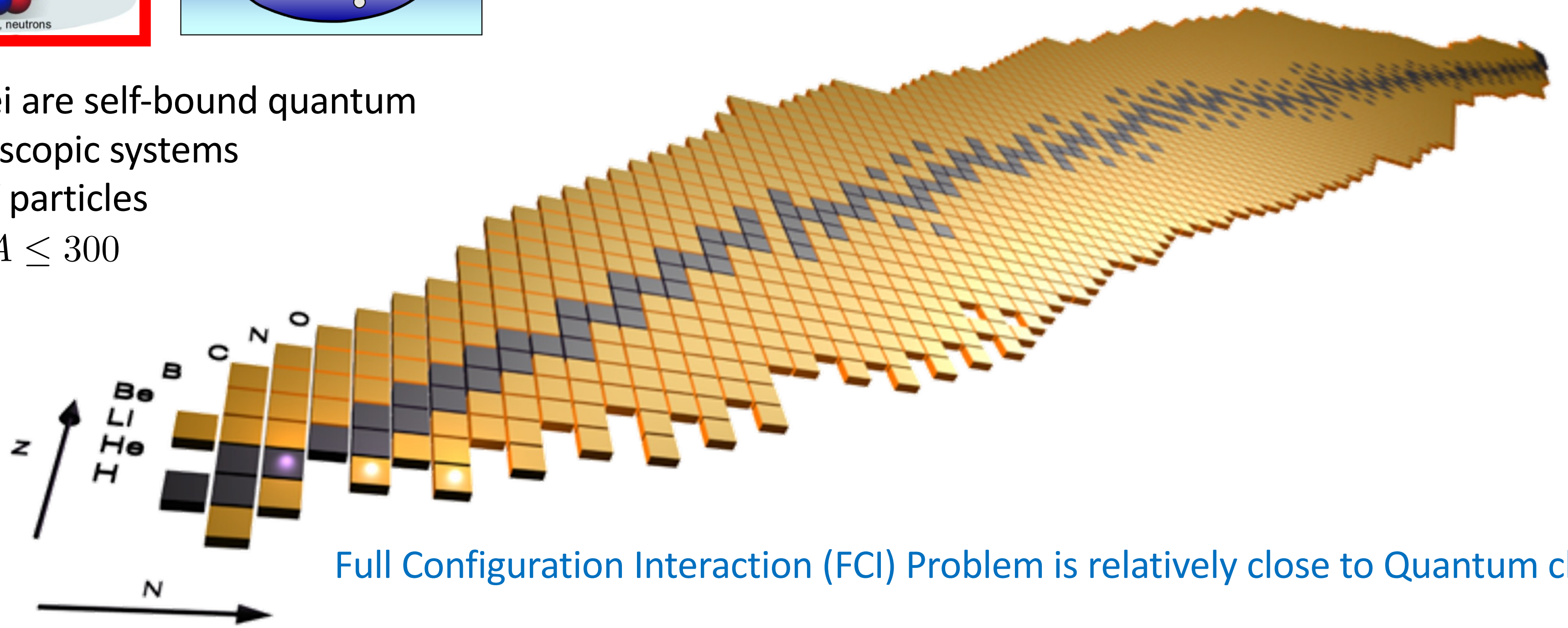
Quantum computing for atomic nuclei

Problematic and challenges

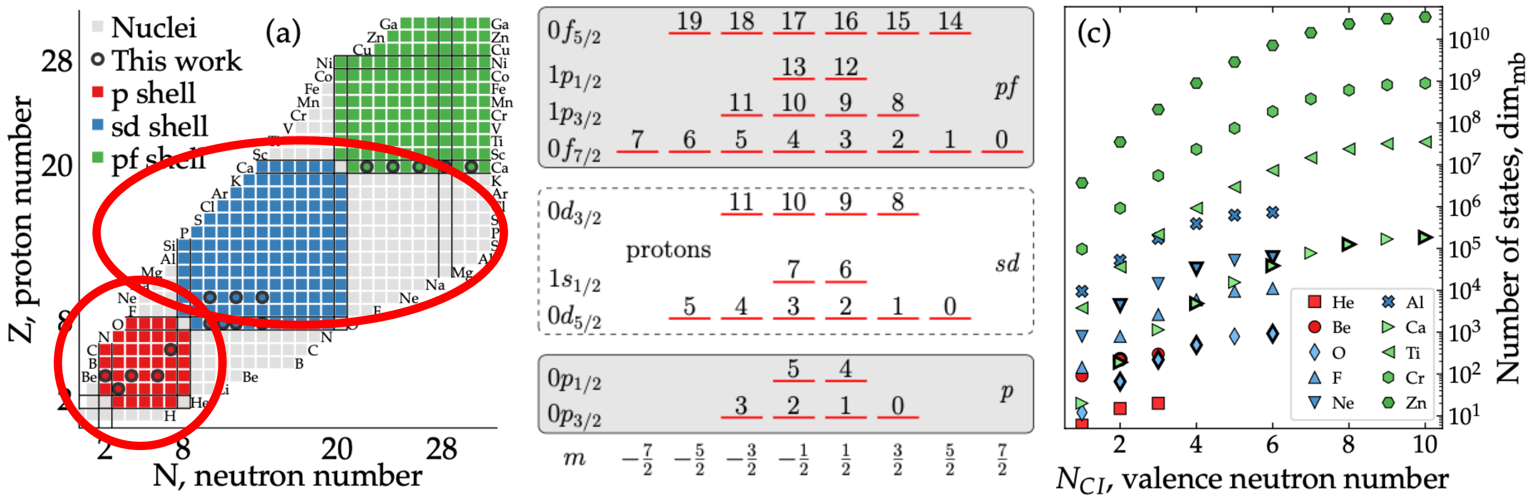




Nuclei are self-bound quantum mesoscopic systems
Nb of particles
$2 \leq A \leq 300$



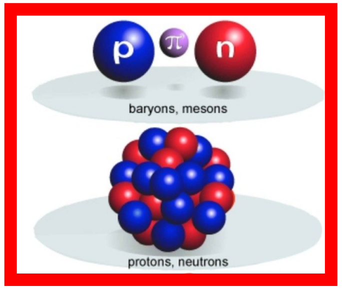
Full Configuration Interaction (FCI) Problem is relatively close to Quantum chemistry

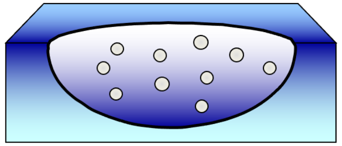


Perez-Obiol et al, Scientific Reports 13 (2023)

Quantum computing for atomic nuclei

Problematic and challenges





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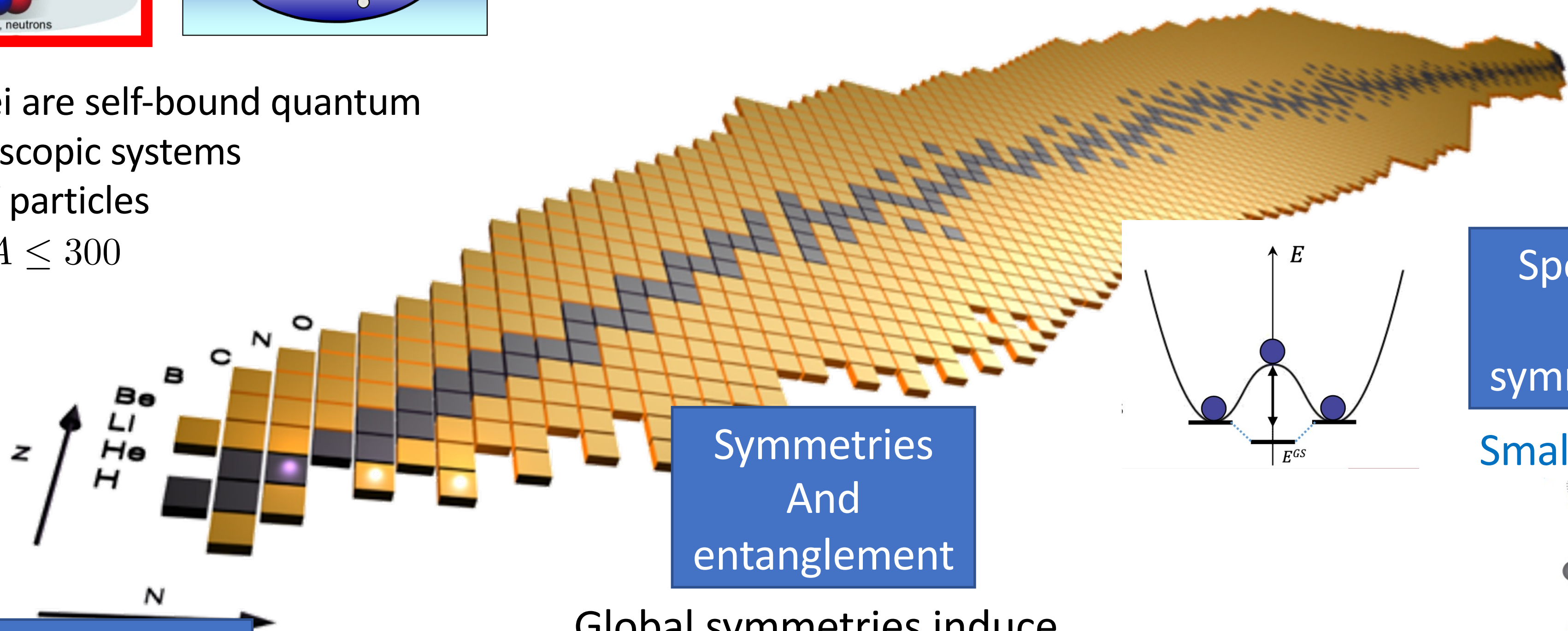
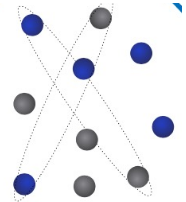


Spontaneous Broken symmetries (SB)

Small superfluid

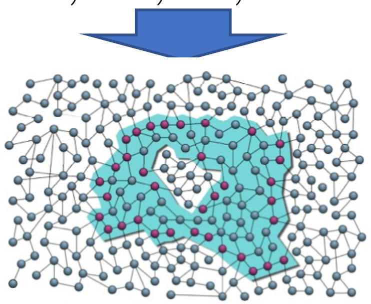


(particle number SB)

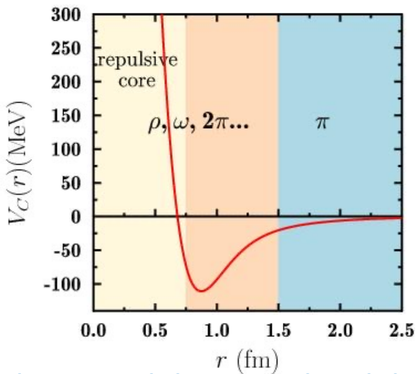
Symmetries And entanglement

Global symmetries induce
All-to-all entanglement

$$S,\ T,\ J,\ \pi$$



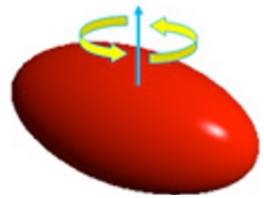
Interaction



The problem is highly non-perturbative

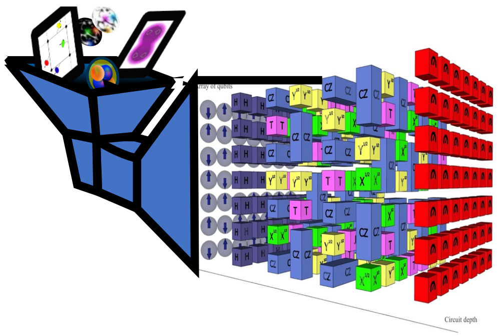
Nuclei are subject to entanglement volume law
(bad candidate for Tensor Network)

Deformation can happen



(rotational invariance SB)

...



Developing variational approaches based on symmetry-breaking (SB)/symmetry restoration (SR)

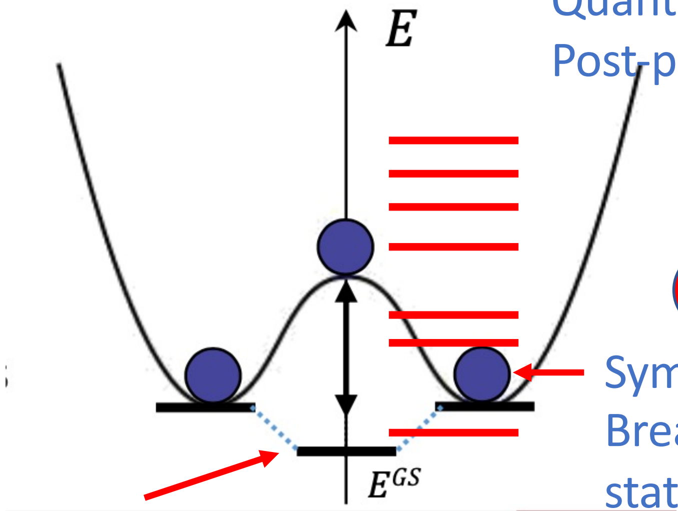
Variational optimization

1 Preparation of SB states on QC

2 Symmetry restoration on QC

3 Post-processing for Improved ground state or excited States (QPE, Quantum Krylov, …)

3 Further Quantum or hybrid Quantum-Classical Post-processing



1 Symmetry Breaking (SB) state

2 Symmetry Restored (SR) state (multi-reference)

D. Lacroix, A. Ruiz Guzman and P. Siwach,
Symmetry breaking/symmetry preserving circuits
and symmetry restoration on quantum computers
EPJA 59 (2023)

Which symmetries ?

Many-Body	Quantum computing
Particle Number	Hamming weight
Parity	Odd/Even number of 1
Total Spin	Permutation Invariance

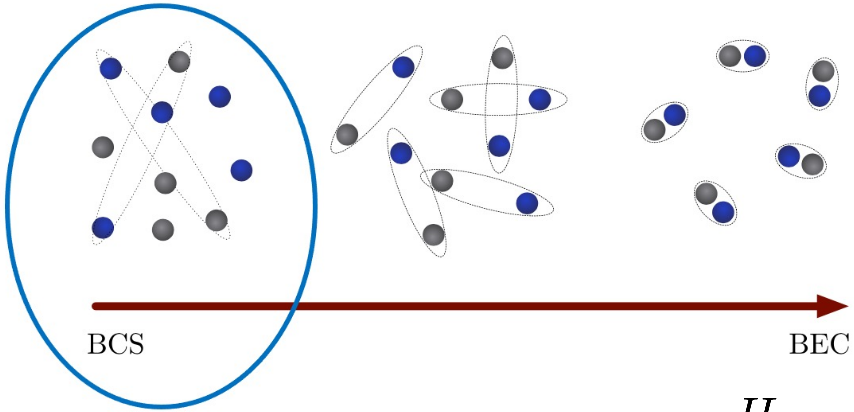
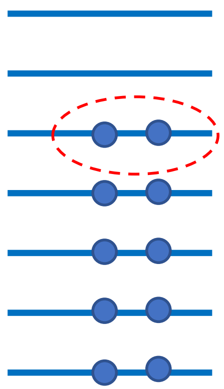


Illustration with small superconductors

Illustration with the Richardson Hamiltonian

$$H_{\mathrm{P}} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

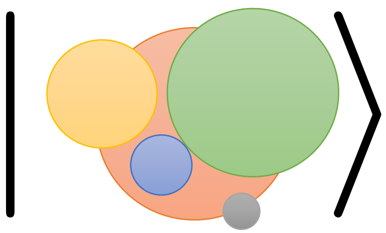


This problem is an archetype of spontaneous symmetry breaking.
An "easy" way to describe it is to break the particle number symmetry, i.e. consider wave-function that mixes different particle number

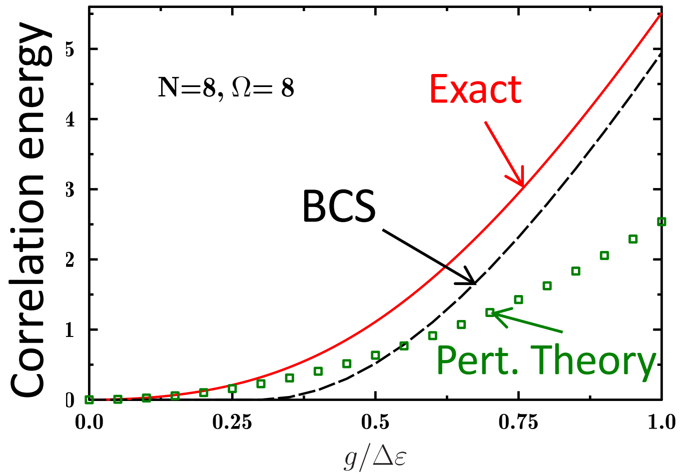
Example $|\Phi_0\rangle = \Pi_i (u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger)|-\rangle$

→ Mixes states with 0, 2, 4, … particles

The particle number - U(1) symmetry) is broken



But ultimately number of Particle should be restored !



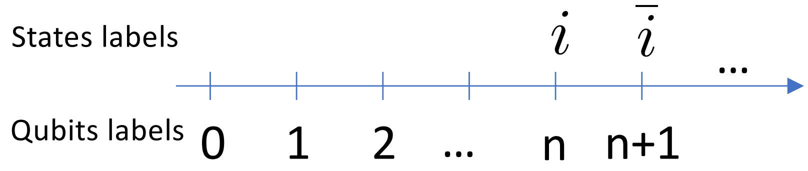
Hamiltonian and initial state

Pairing Hamiltonian

$$H_{\mathrm{P}} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

Jordan-Wigner transfo: $\frac{1}{2}(I_i - Z_i)$

State ordering is important !

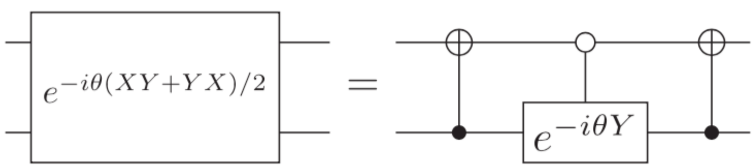


$$a_i^\dagger a_{\bar{i}}^\dagger \Rightarrow Q_n^+ Q_{n+1}^+$$

Initial (symmetry breaking) state preparation

$$|\Psi\rangle = \exp\left\{ - \sum_{i>0} \varphi_i \left(a_i^\dagger a_{\bar{i}}^\dagger - a_{\bar{i}} a_i \right) \right\} |0\rangle \quad \xrightarrow{\varphi_i = \varphi} \quad |\Psi\rangle = \prod_{n>0} e^{i\varphi(X_n Y_{n+1} + Y_n X_{n+1})/2} |-\rangle$$

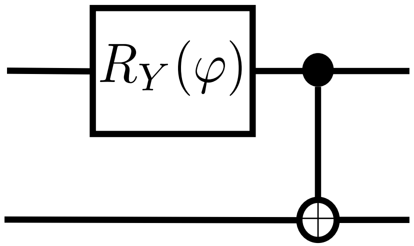
Equivalent universal gate on pairs

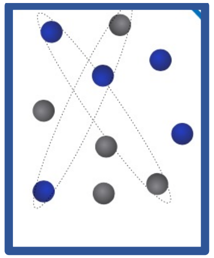


Zhang Jiang et al,
Phys. Rev. Applied 9, 044036 (2018).

Simplified circuit (generalized Bell state)

$$|\Psi\rangle = \prod_n \left[\cos\left(\frac{\varphi}{2}\right) I_n \otimes I_{n+1} + \sin\left(\frac{\varphi}{2}\right) Q_n^+ Q_{n+1}^+ \right] |-\rangle$$

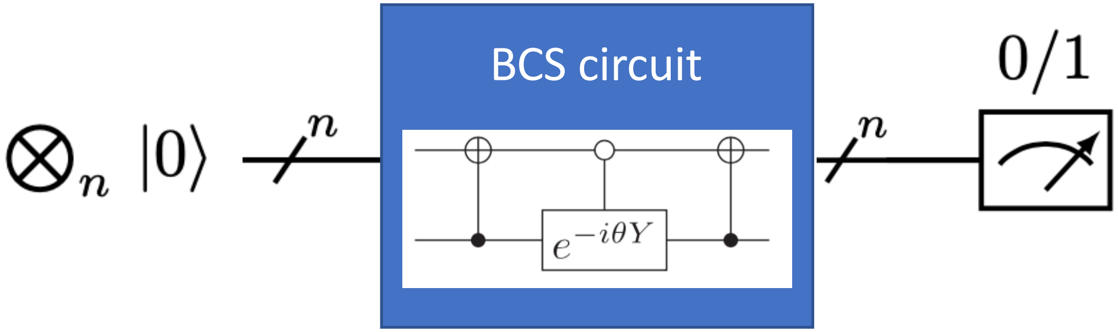




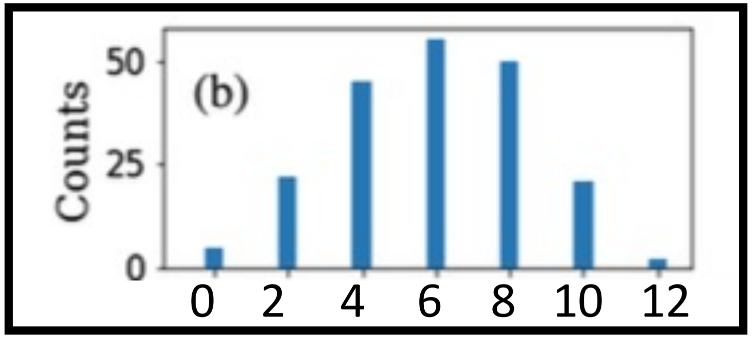
Quantum computing for atomic nuclei

Illustration for small superfluids

Superfluidity can be described by breaking particle number

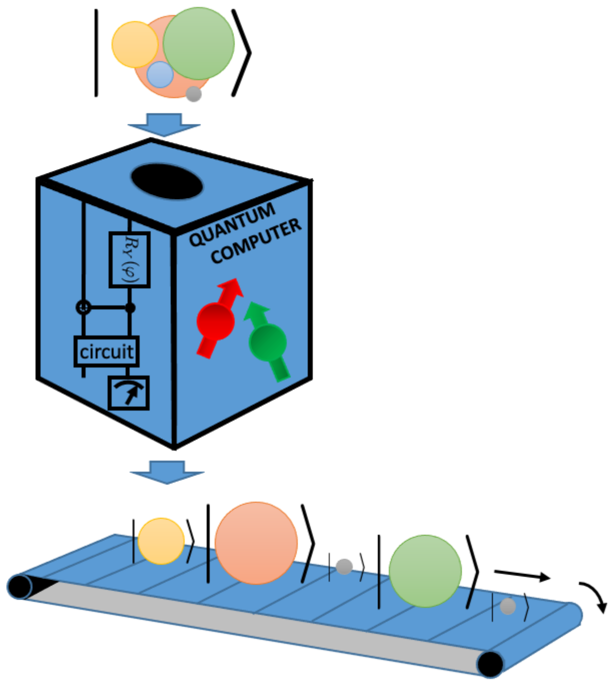


Example of mixing for 12 qubits (with qiskit)



Projection on particle number

$$|\Psi\rangle = \sum_N c_N |N\rangle \Rightarrow |N\rangle$$



For 2 qubits

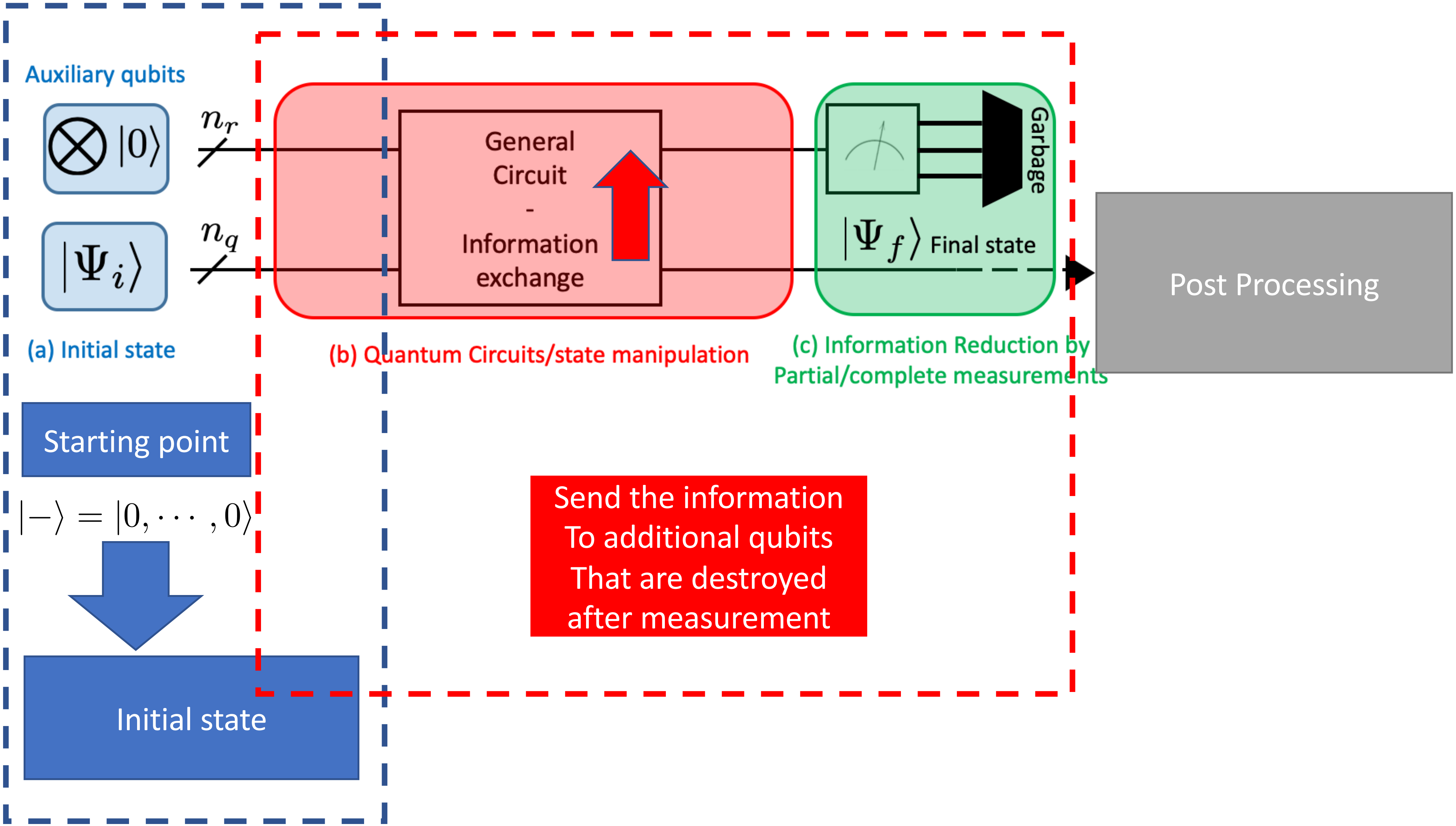
$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$|N=0\rangle \qquad \propto |N=1\rangle \qquad |N=2\rangle$$

➡ A possible way to perform the projection is to use The Quantum-Phase-Estimation method with *N* itself

D. Lacroix, "*Symmetry-Assisted Preparation of Entangled Many-Body States on a Quantum Computer*", PRL 125, 230502 (2020).

Non-destructive counting on a quantum computer



D. Lacroix, "*Symmetry-Assisted Preparation of Entangled Many-Body States on a Quantum Computer*", PRL 125, 230502 (2020).

The quantum-Phase estimation (QPE) algorithm

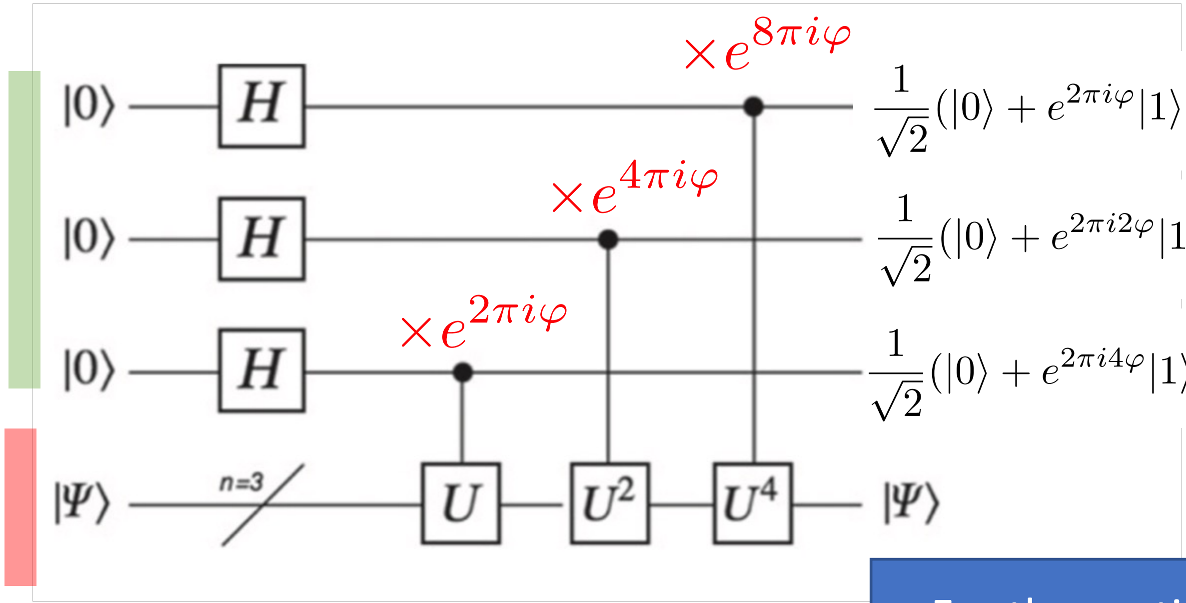
For known eigenvalue problems

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$

nr Register qubits

Working qubits



QFT $^{-1}$ $\rightarrow$ $|2^{n_r}\varphi\rangle \otimes |\Psi\rangle$

General Case

$$|\Psi\rangle = \sum_k \alpha_k|\phi_k\rangle \xrightarrow{\text{QPE}} \sum_k \alpha_k|\theta_k 2^{n_r}\rangle \otimes |\phi_k\rangle$$

register eigenstate

For the particle number projection

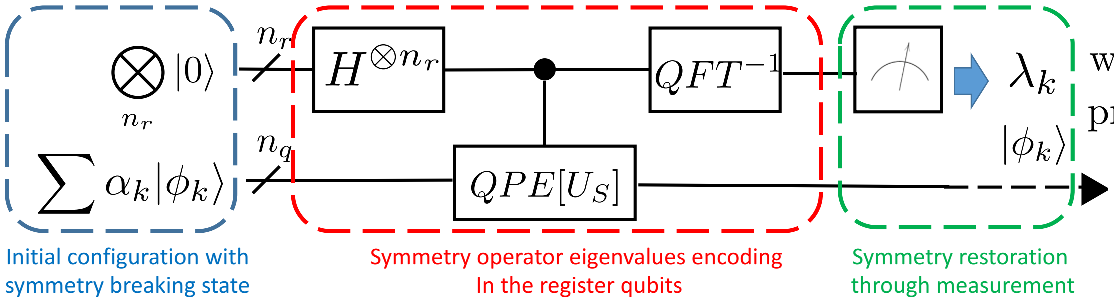
$$U = U_N = e^{2\pi i\frac{N}{2^{n_r}}} \quad \text{with} \quad N = \frac{1}{2}\sum_i (I_i - Z_i)$$

Assume eigenvalues $\{0, 1, \cdots, A\}$

Constraint: $0 \leq \frac{A}{2^{n_r}} < 1$ then $\frac{A}{2^{n_r}} = 0.a_1 \cdots a_{n_r-1}$

If I measure given binary number in the ancillary qubit. After measurement, I have the projection on the associated particle number component

Eigenvalues-Ground state and excited states



with prob. $|\alpha_k|^2$

Measurement

Example of an event:

$$|011\cdots010\rangle^{(\lambda)} \otimes |\phi_{A^{(\lambda)}}\rangle$$

$$= A^{(\lambda)}$$

BCS/HFB state

$$|\Psi\rangle = \prod_n \left[\cos\left(\frac{\varphi}{2}\right) I_n \otimes I_{n+1} + \sin\left(\frac{\varphi}{2}\right) Q_n^+ Q_{n+1}^+\right] |-\rangle$$

Measurement

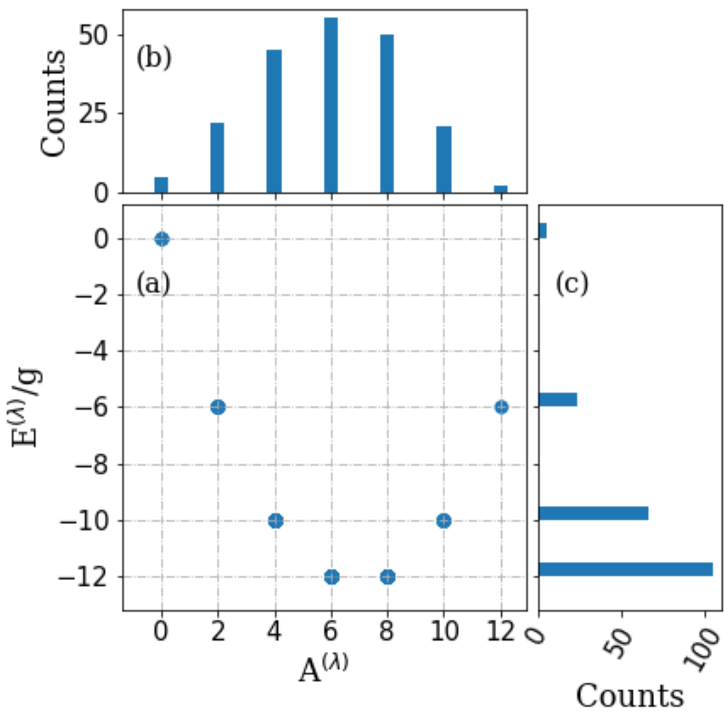
Projected BCS or with varying number Of particles

Degenerate case

$$H_{\rm P} = -g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

$$\langle \phi_{A^{(\lambda)}} | H | \phi_{A^{(\lambda)}} \rangle$$

6 pairs



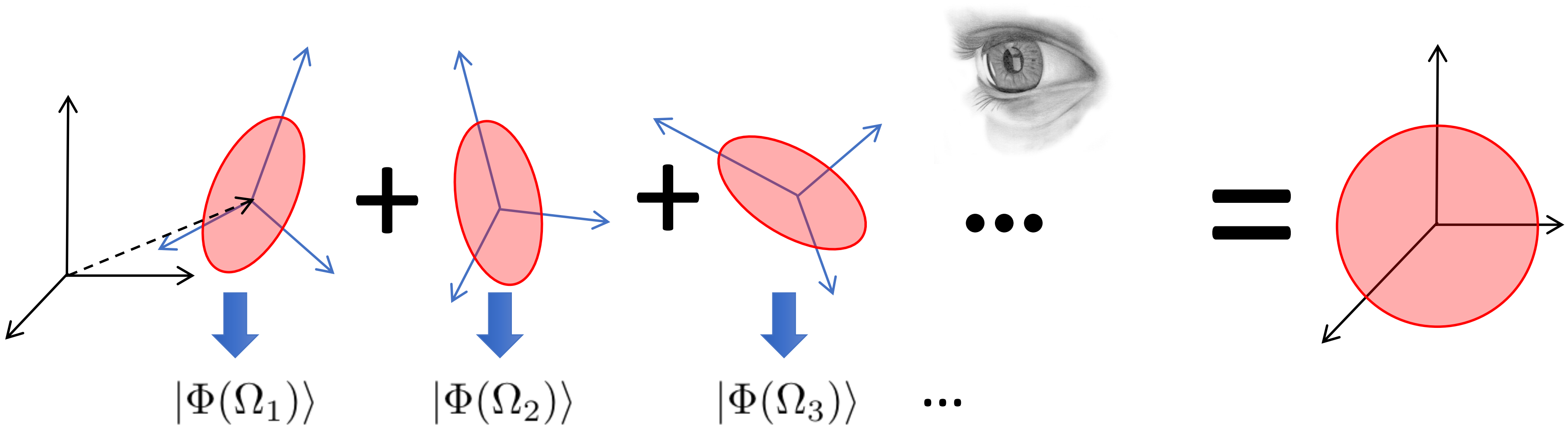
Exact solution

$$E/g = -\frac{1}{4}(A-\nu)(2n_q - A - \nu - 2).$$

H was encoded on the full Fock space with $A < n_q$

For the degenerate case, this should give the exact solution

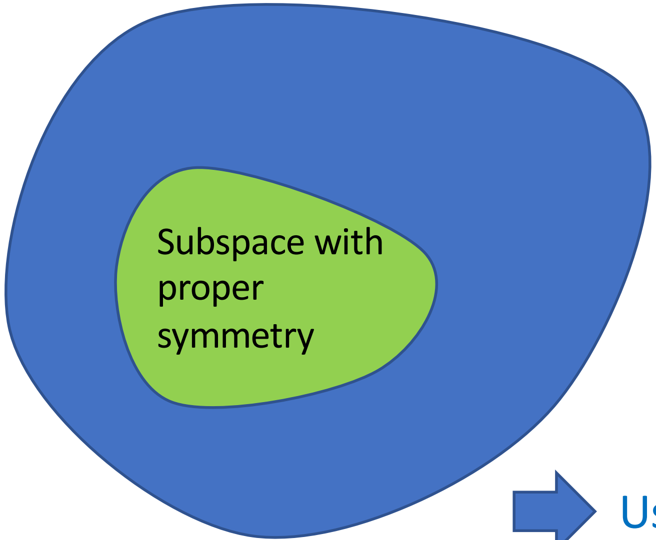
More on symmetry restoration techniques on quantum computer



D. Lacroix, A. Ruiz Guzman and P. Siwach, Symmetry breaking/symmetry preserving circuits and symmetry restoration on quantum computers, EPJA 59 (2023)

Exploration of different methods for the symmetry restoration

Complete Hilbert space



Systematic Exploration of Phase-Estimation based methods for symmetry restoration (Kitaev method, Rodeo algorithms)

Eur. Phys. J. A (2023) 59:3
https://doi.org/10.1140/epja/s10050-022-00911-7
THE EUROPEAN PHYSICAL JOURNAL A
Regular Article - Theoretical Physics

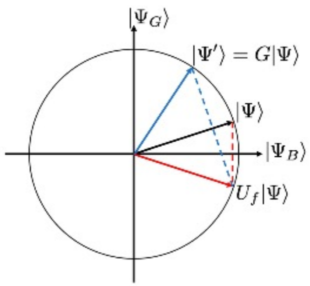
Symmetry breaking/symmetry preserving circuits and symmetry restoration on quantum computers

A quantum many-body perspective

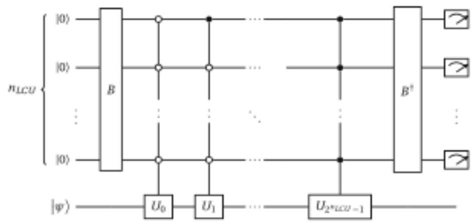
Denis Lacroix[1,a], Edgar Andres Ruiz Guzman[1,b], Pooja Siwach[2,c]

Use Oracle's and Grover-based methods for projection onto a subspace

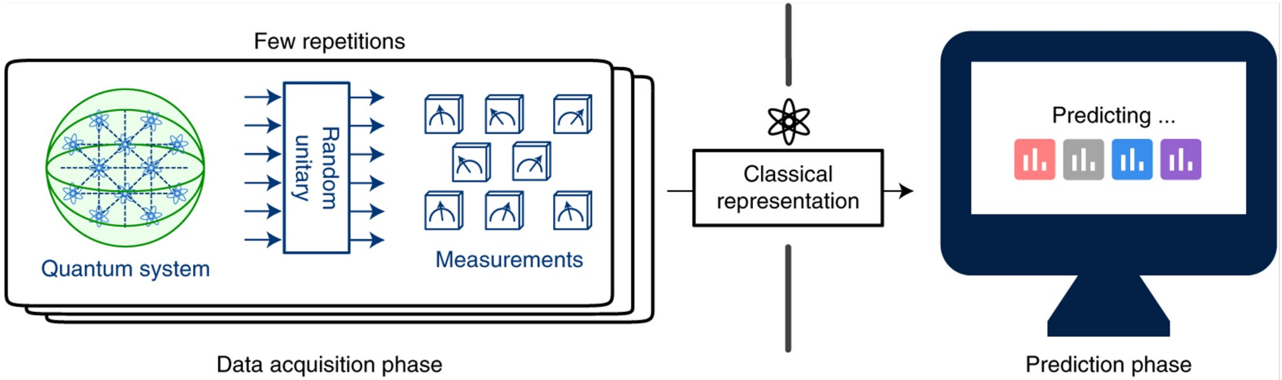
Grover and Oracle



Linear Combination of Unitaries



Use quantum tomography techniques (Classical Shadow method)



H.-Y. Huang, R. Kueng and J. Preskill; Nat. Phys. 16, 1050 (2020)

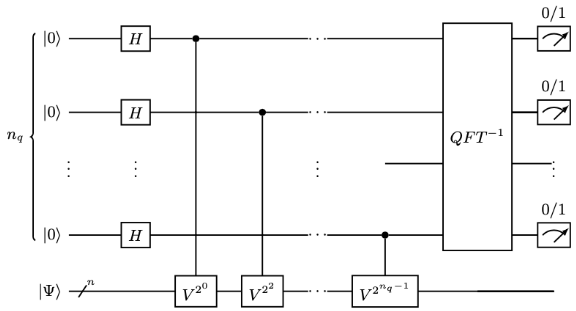
Restoring broken symmetries using quantum search "oracles"

Edgar Andres Ruiz Guzman and Denis Lacroix

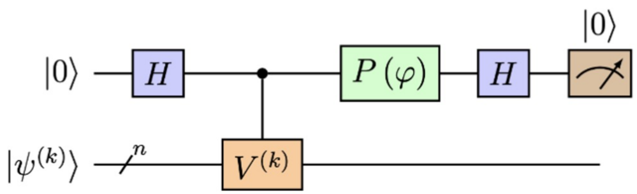
Phys. Rev. C **107**, 034310 (2023) - Published 16 March 2023

E.A. Ruiz Guzman and DL, Eur. J. Phys. A 60 (2024)

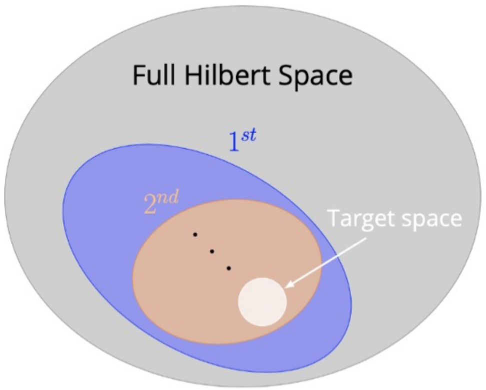
Standard Quantum Phase estimation



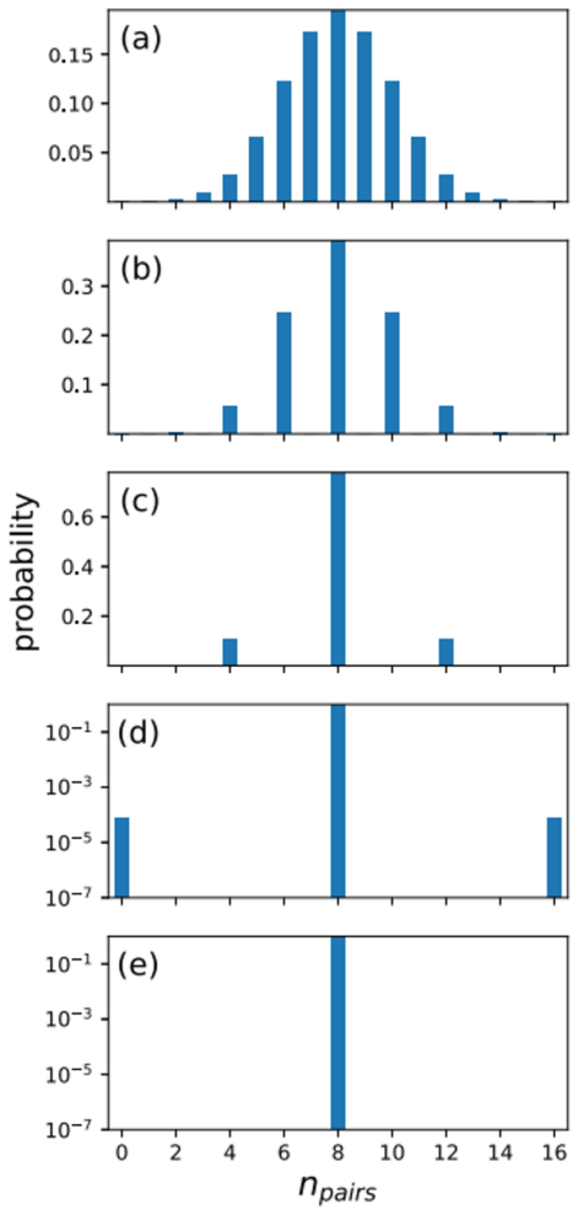
Iterative Quantum Phase estimation



$$\hat{V}^{(k)} = e^{i\phi_k \hat{N}} \qquad \phi_k = \frac{\pi}{2^k}$$

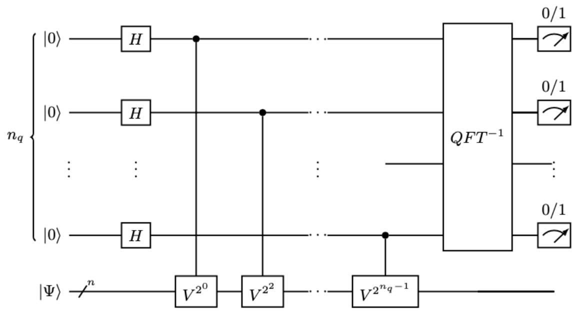


16 qubits, N = 8

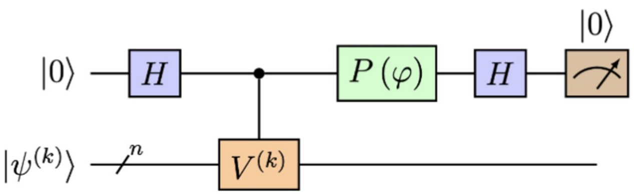


Systematic of QPE-based methods

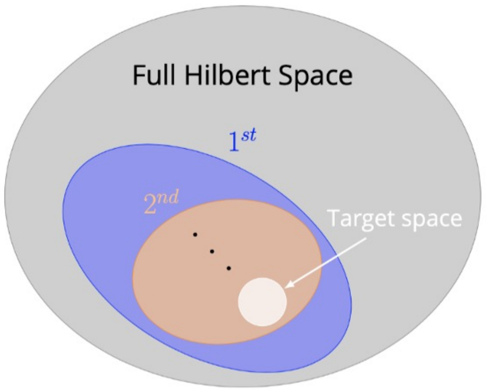
Standard Quantum Phase estimation



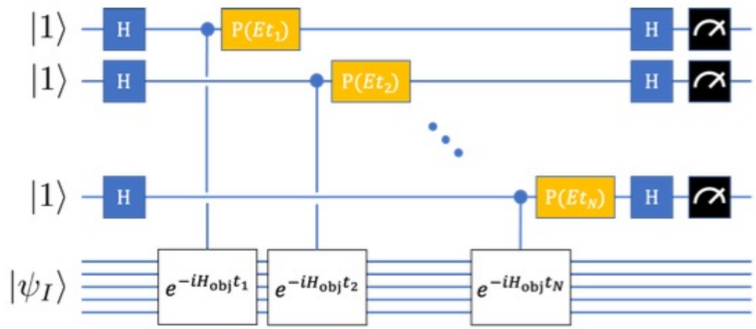
Iterative Quantum Phase estimation



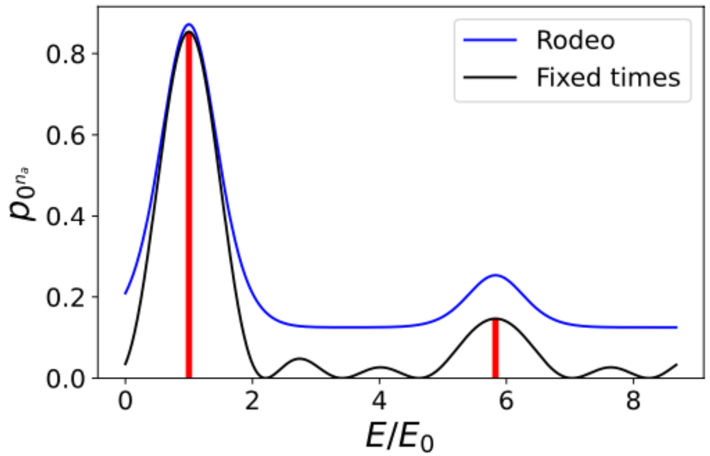
$$\hat{V}^{(k)} = e^{i\phi_k \hat{N}} \qquad \phi_k = \frac{\pi}{2^k}$$



Iterative Quantum Phase estimation
+ random Gaussian time (Rodeo algorithm)



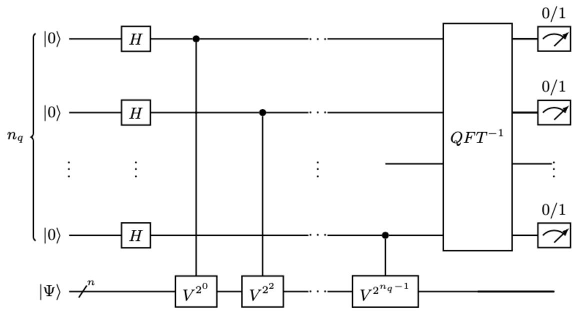
K. Choi et al., Rodeo Algorithm for Quantum Computing, Phys. Rev. Lett. 127, 040505 (2021).



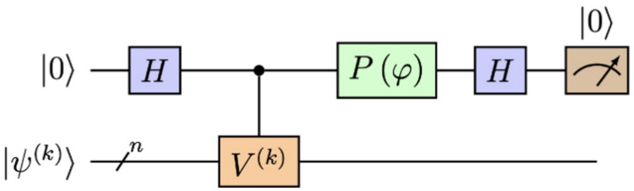
Ayral, Besserve, Lacroix, Ruiz Guzman, EPJA 59 (2023)

Systematic of QPE-based methods

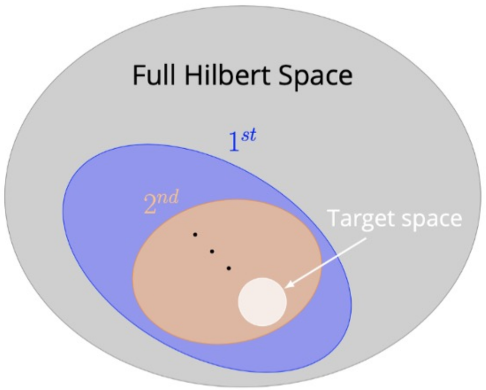
Standard Quantum Phase estimation



Iterative Quantum Phase estimation

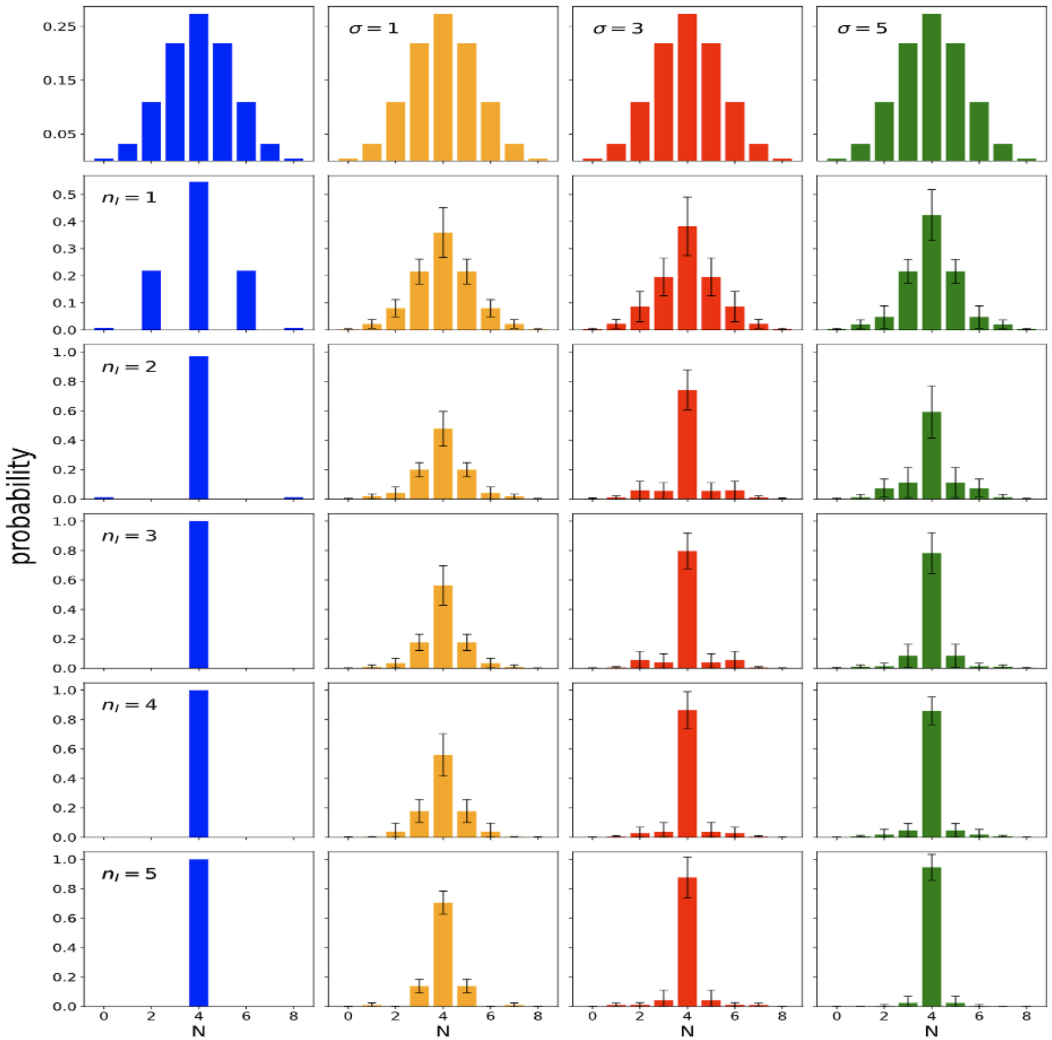


$$\hat{V}^{(k)} = e^{i\phi_k \hat{N}} \qquad \phi_k = \frac{\pi}{2^k}$$



Iterative Quantum Phase estimation + random Gaussian time (Rodeo algorithm)

Iterative QPE   Rodeo algorithm with different resolution

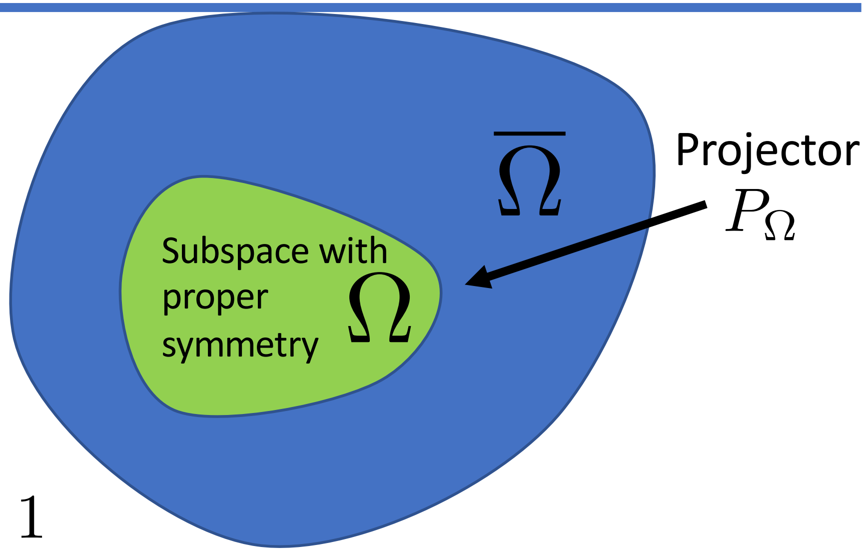


Symmetry restoration using Projection operators and Oracles

Grover Classification operator

$$\hat{U}_f|k\rangle = (-1)^x \text{ with } x \begin{cases} 0 & if \;\; |k\rangle \in \bar{\Omega} \\ 1 & if \;\; |k\rangle \in \Omega \end{cases}$$

Lov K. Grover, Phys. Rev. Lett. **79**, 325 (1997)

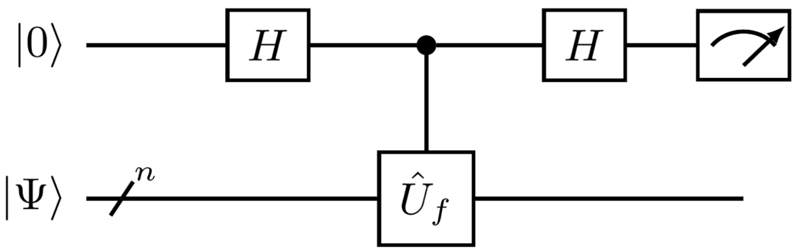


Assume we are able to encode the projector

$$P_\Omega \Rightarrow U_f = +1P_\Omega - 1(1 - P_\Omega) = 2P_\Omega - 1$$

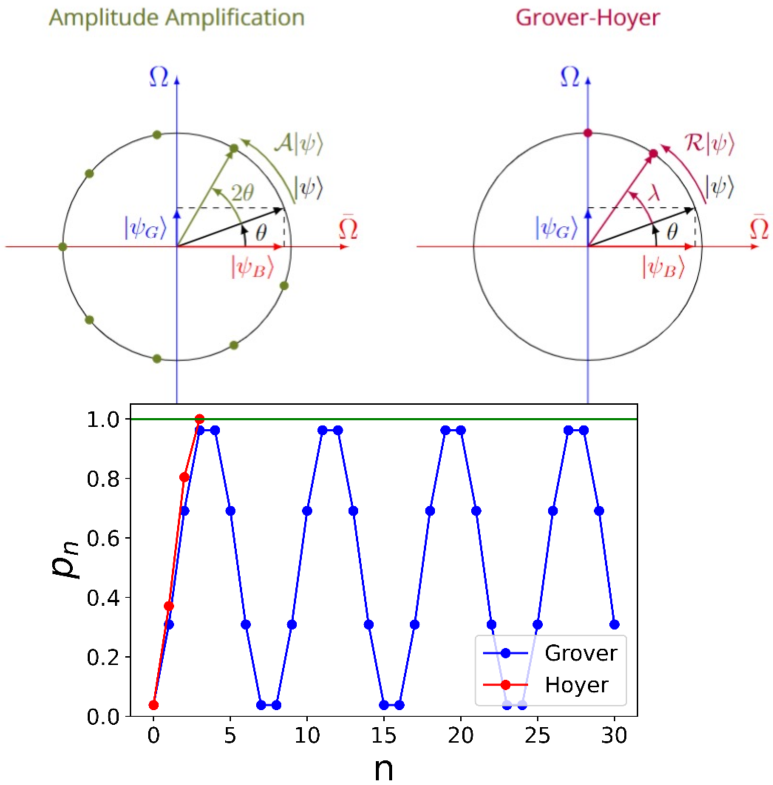
Methods based on projectors

Oracle + Hadamard test



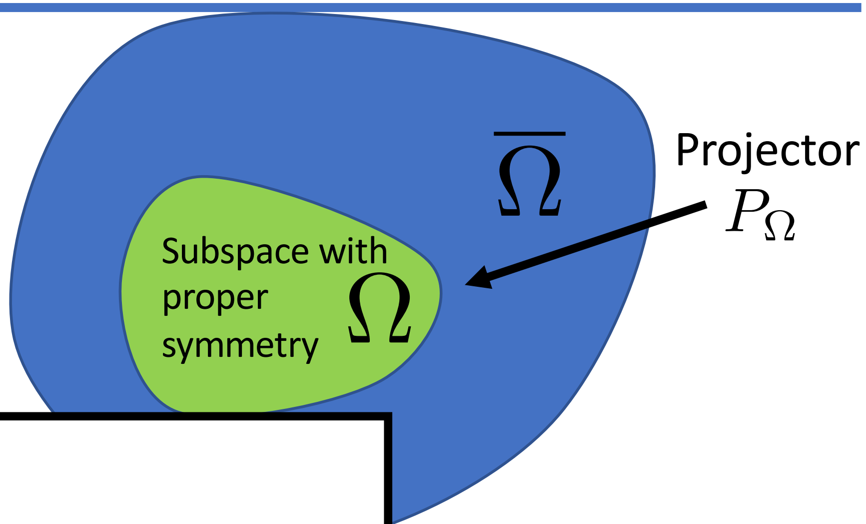
$$\frac{1}{2}\{|0\rangle \otimes [I + \hat{U}_f]|\Psi\rangle + |1\rangle \otimes [I - \hat{U}_f]|\Psi\rangle\} = |0\rangle|\Psi_B\rangle + |1\rangle|\Psi_G\rangle$$

Grover technique



E. A. Ruiz Guzman and D. Lacroix, Phys. Rev. **C 107**, 034310 (2023)

Symmetry restoration using Projection operators and Oracles



Grover Classification operator

$$\hat{U}_f|k\rangle = (-1)^x \text{ with } x \begin{cases} 0 & if\ |k\rangle \in \overline{\Omega} \\ 1 & if\ |k\rangle \in \Omega \end{cases}$$

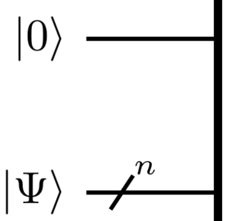
Lov K. Grover, Phys. Rev. Lett. **79**, 325 (1997)

Assume w

P_Ω

Methods

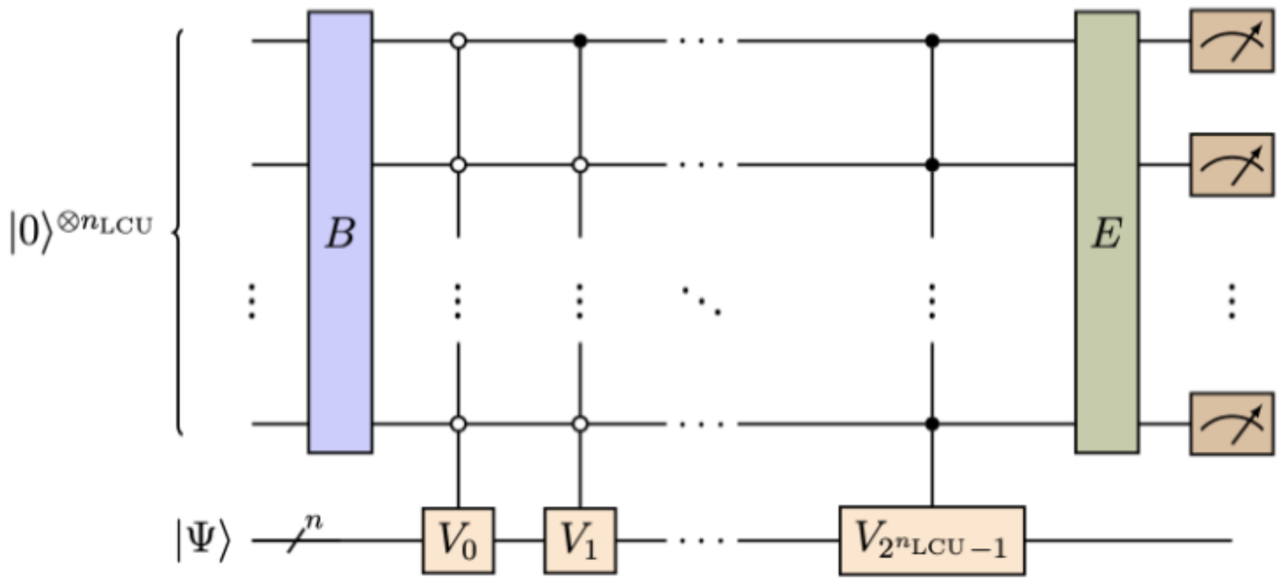
Oracle + H

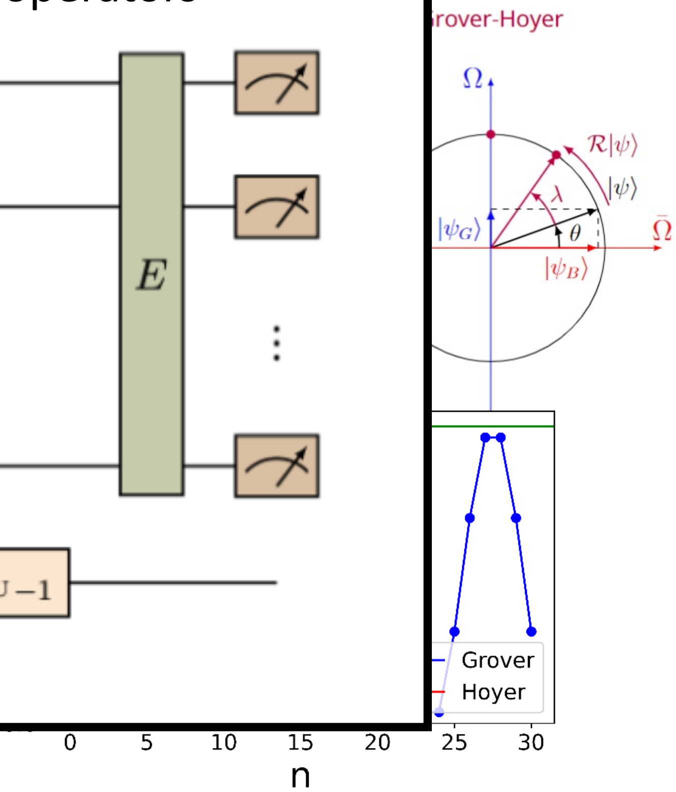


$\frac{1}{2}\{|0\rangle \otimes [I + \hat{U}$

Practical implementation of projectors

$$P_N = \frac{1}{n+1}\sum_{k=0}^{n} e^{\frac{2\pi i k(\hat{N}-N)}{n+1}}$$ = sum of unitary operators





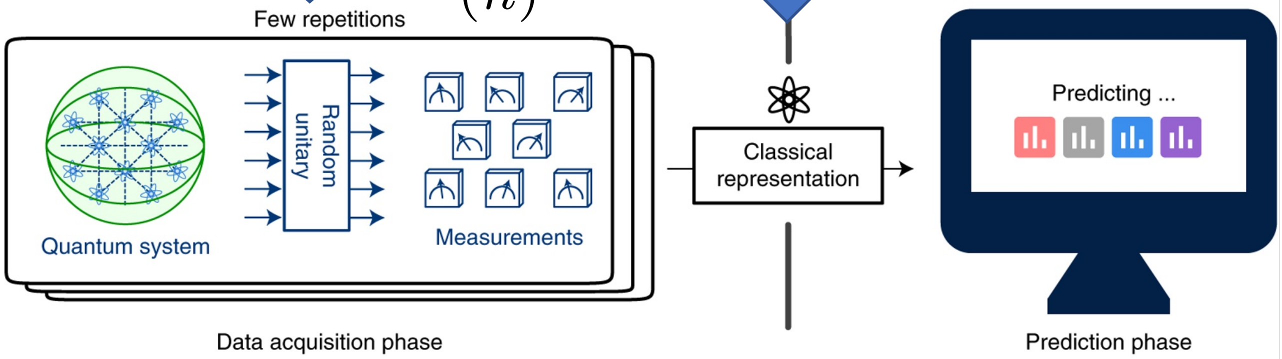
E. A. Ruiz Guzman and D. Lacroix, Phys. Rev. **C 107**, 034310 (2023)

Symmetry restoration by quantum tomography

Classical shadow technique

Random rotations of the computational basis $U_{(n)}$

Storing of pseudo classical bits $\mathbf{b}^{(n)} = b^{(n)}_{q-1} \cdots b_0$



H.-Y. Huang, R. Kueng and J. Preskill; Nat. Phys. 16, 1050 (2020)

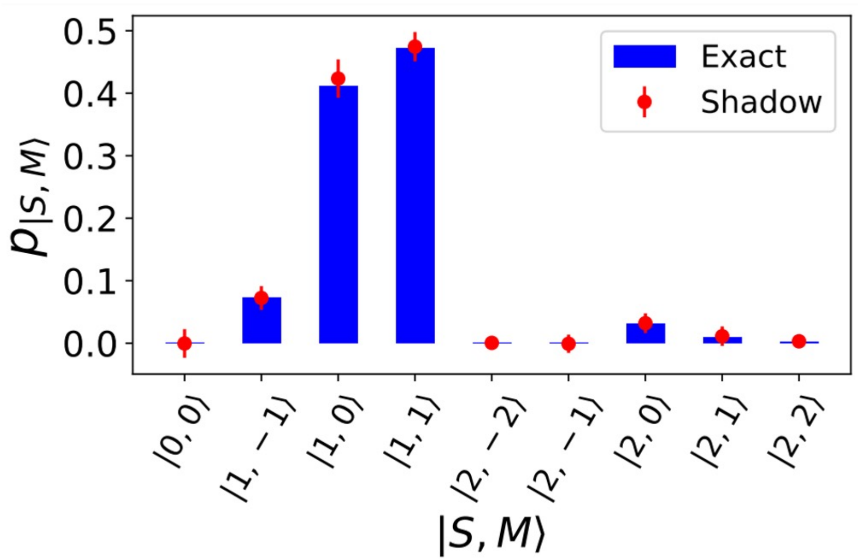
Snapshot

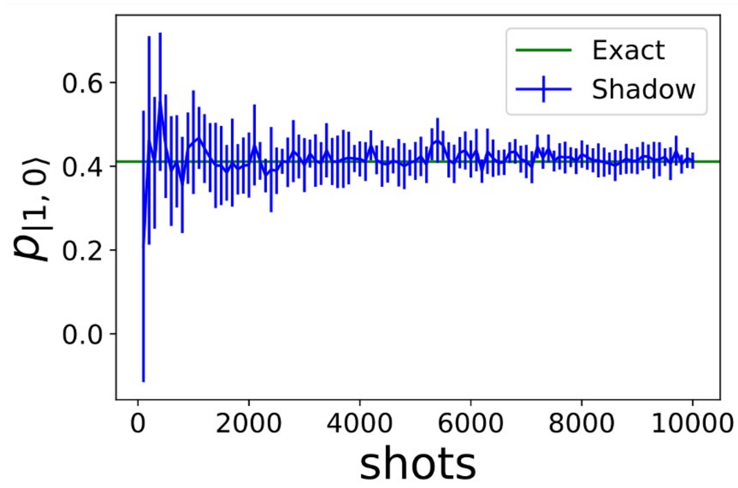
$$\rho^{(n)} = |\mathbf{b}^{(n)}\rangle\langle\mathbf{b}^{(n)}|$$

Inverse transformation average

$$\langle O \rangle = \frac{1}{N_{\rm evt}} \sum_n \mathrm{Tr}\left(O U^\dagger_{(n)} \rho^{(n)} U_{(n)} \right)$$

+ optimization on the convergence





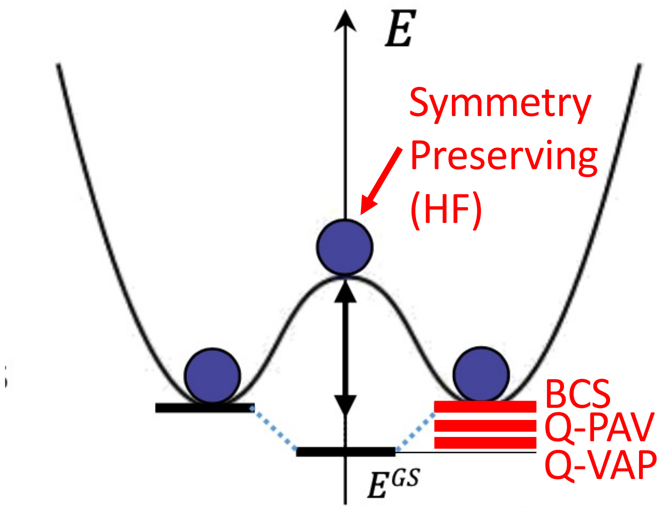
E.A. Ruiz Guzman and D. Lacroix, EPJA 60 (2024)

Coming back to our superconducting problem
Combining projection with variational method

Possible optimization schemes

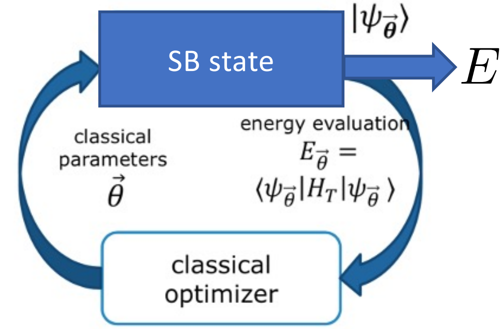
Pair occupation are now encoded

Variational
Symmetry-Breaking ansatz
$$|\Psi(\{\theta_p\})\rangle = \bigotimes_{p=1}^{N-1} [\sin(\theta_p)|0_p\rangle + \cos(\theta_p)|1_p\rangle]$$

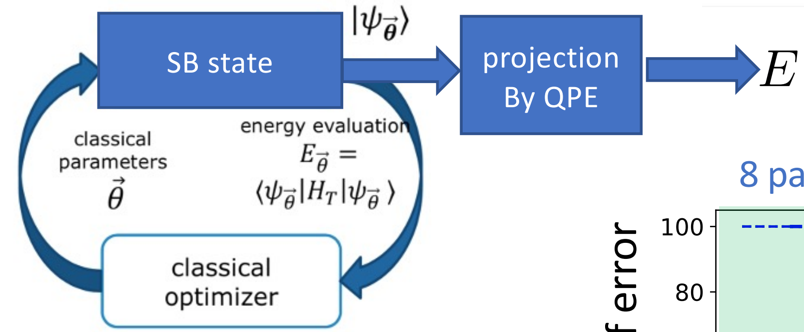


Quantum-Classical optimizers

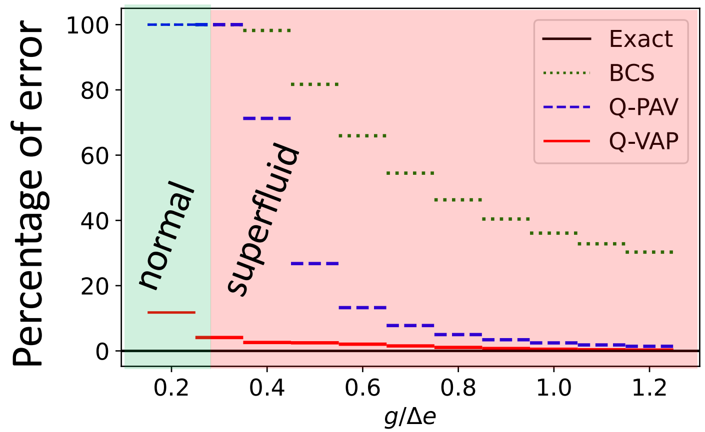
- Standard BCS theory



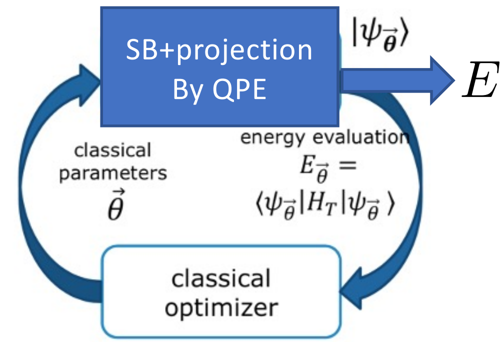
- Project after optimization
  Q-PAV: Quantum Projection After Variation



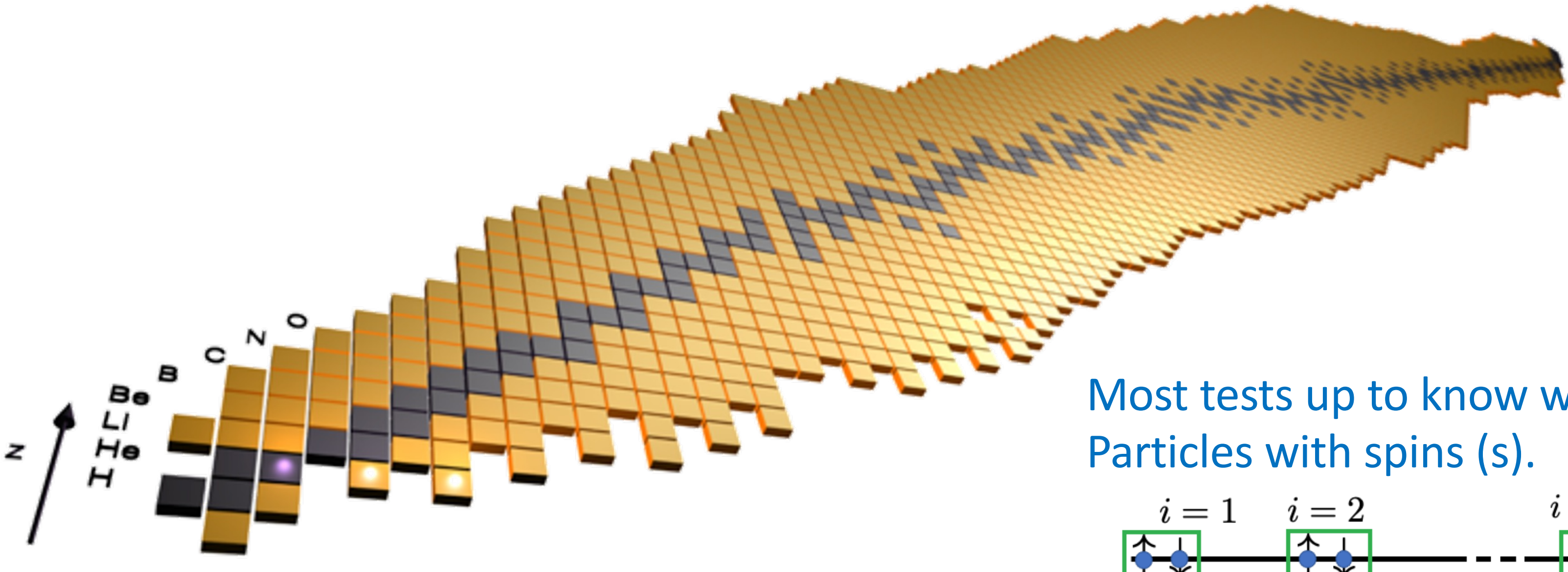
8 particles on 8 equidistant levels



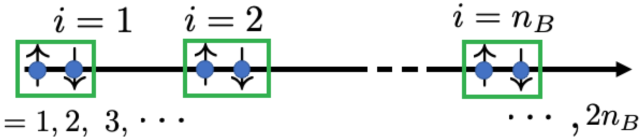
- The optimization is made on the Symmetry restored state.
  Q-VAP: Quantum Variation After Projection



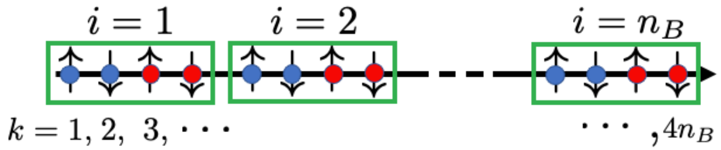
Is the breaking of symmetries always a good idea?



Most tests up to know were made on Particles with spins (s).



But nuclei have both spin (s) and isospin (t) (neutron/proton)



- This increases the number of qubits

$$S_z, \ S^2, \ \pi$$

- This increases the number of symmetries that could be broken

$$S_z, \ S^2, \ T_z, \ T^2, \ \pi$$

Symmetry-breaking states become extremely hard to control
Symmetry restoration becomes very demanding

J. Zhang, PhD thesis (2025).

Use of adaptative methods

And try to control symmetry breaking

Iterative construction of the ansatz

Grimsley, et al, Nat. Commun. 10 (2019)

- Start from a state $|\Psi_0\rangle = |n=0\rangle$

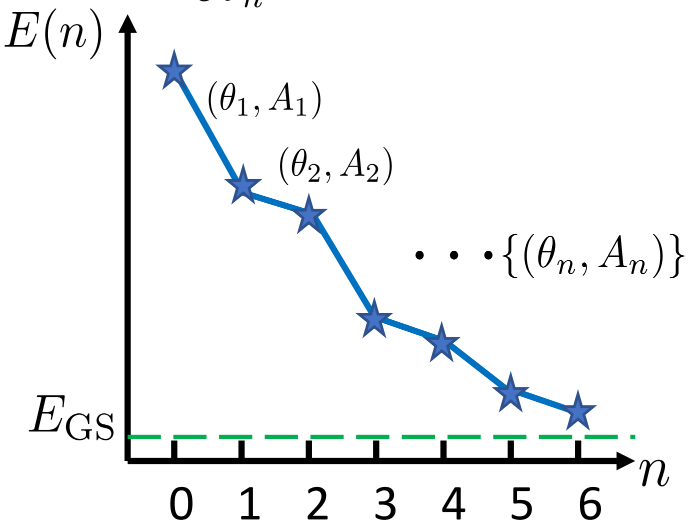
- Built iteratively the ansatz such as:

$$|n\rangle = e^{i\theta_n A_n}|n-1\rangle = \prod_{k=1}^{n} e^{i\theta_k A_k}|0\rangle$$

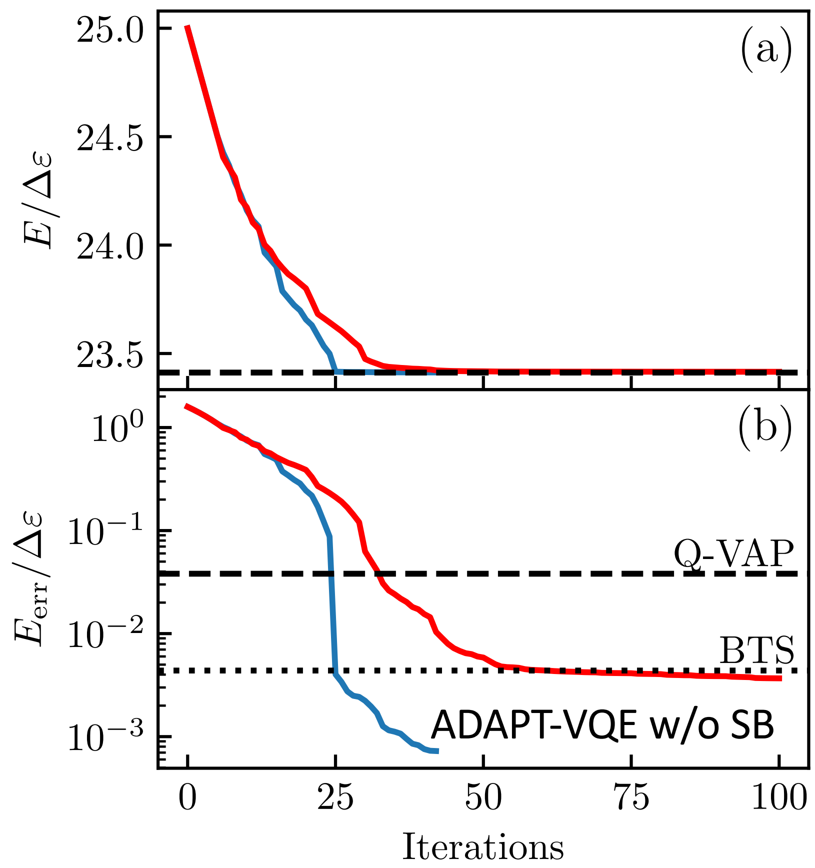
$$A_n \in \{O_1, \cdots, O_\Omega\}$$

Such that

$$\frac{\partial E(n)}{\partial \theta_n} = i\langle n|\,[H, A_n]\,|n\rangle \text{ is maximum}$$

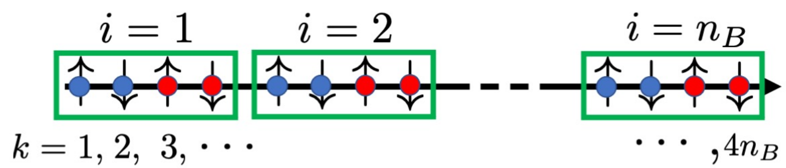


ADAPT-VQE applied to the Superfluid problems: only spins



J. Zhang, D. Lacroix, and Y. Beaujeault-Taudière, arXiv:2408.17294

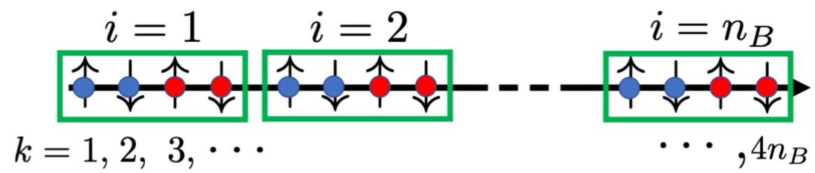
Extension to spin and isospin



Is breaking symmetries always a good idea?

Extension to the proton-neutron pairing Hamiltonian problem

$$H = \sum_{i=1}^{n_B} \left[\varepsilon_{i,n}(\nu_i^\dagger \nu_i + \nu_{\bar{i}}^\dagger \nu_{\bar{i}}) + \varepsilon_{i,p}(\pi_i^\dagger \pi_i + \pi_{\bar{i}}^\dagger \pi_{\bar{i}}) \right] - \sum_{T_z=-1,0,1} g_V(T_z) \mathcal{P}_{T_z}^\dagger \mathcal{P}_{T_z} - \sum_{S_z=-1,0,1} g_S(S_z) \mathcal{D}_{S_z}^\dagger \mathcal{D}_{S_z}.$$



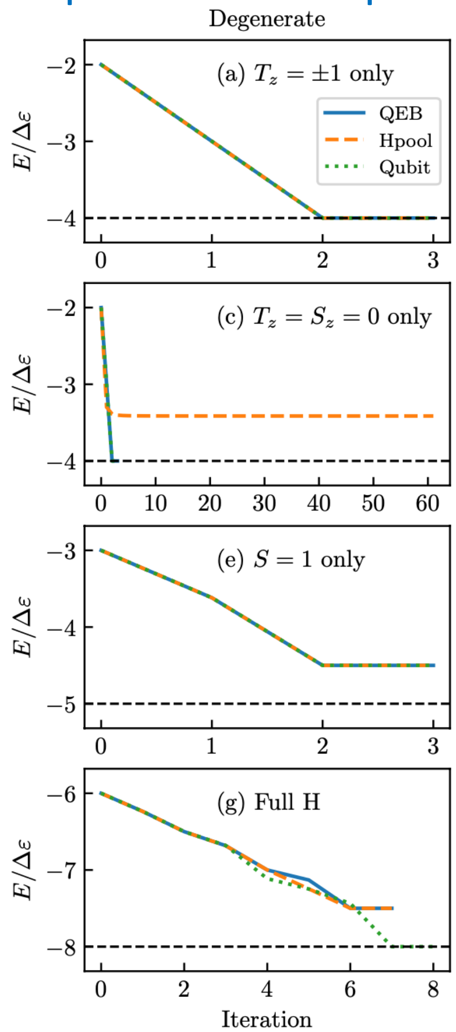
Different Hamiltonian limit

Case \ S_z/T_z	Isoscalar −1	Isoscalar 0	Isoscalar 1	Isovector −1	Isovector 0	Isovector 1
1				✓		✓
2		✓			✓	
3				✓	✓	✓
4	✓	✓	✓	✓	✓	✓

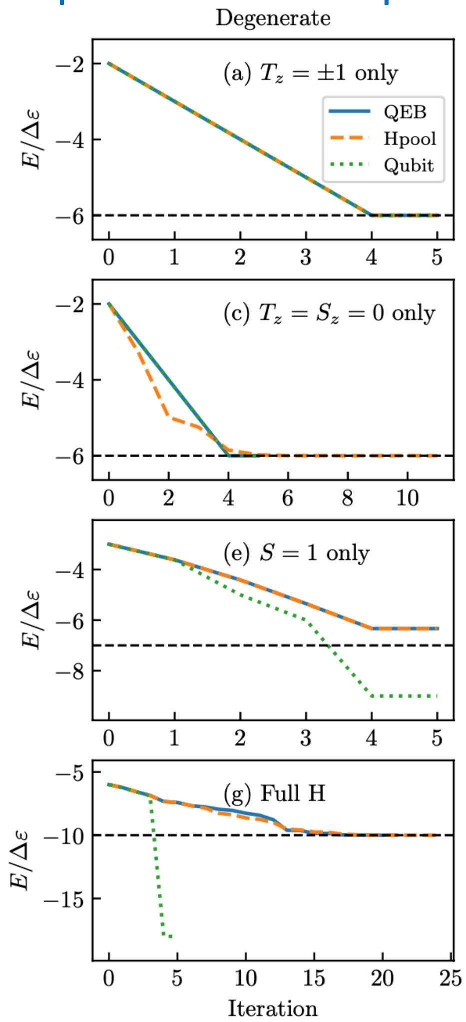
Different operator pool in ADAPT-VQE breaking or not symmetries

	Particle number	Seniority	Parity
H-pool	✓	✓	✓
QEB-pool	✓	×	✓
Qubit-pool	×	×	✓

4 particles on 8 qubits



4 particles on 12 qubits



J. Zhang, DL, and Y. Beaujeault-Taudière, arXiv:2408.17294

Is breaking symmetries always a good idea?

Extension to the proton-neutron pairing Hamiltonian problem

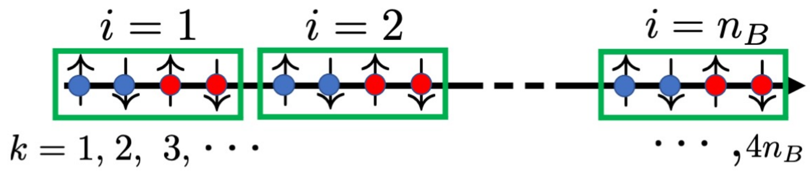
$$H = \sum_{i=1}^{n_B} \left[\varepsilon_{i,n}(\nu_i^\dagger \nu_i + \nu_{\bar{i}}^\dagger \nu_{\bar{i}}) + \varepsilon_{i,p}(\pi_i^\dagger \pi_i + \pi_{\bar{i}}^\dagger \pi_{\bar{i}}) \right]$$
$$- \sum_{T_z=-1,0,1} g_V(T_z) \mathcal{P}_{T_z}^\dagger \mathcal{P}_{T_z}$$
$$- \sum_{S_z=-1,0,1} g_S(S_z) \mathcal{D}_{S_z}^\dagger \mathcal{D}_{S_z}.$$

Different Hamiltonian limit

Case \ S_z/T_z	Isoscalar −1	Isoscalar 0	Isoscalar 1	Isovector −1	Isovector 0	Isovector 1
1				✓		✓
2		✓			✓	
3				✓	✓	✓
4	✓	✓	✓	✓	✓	✓

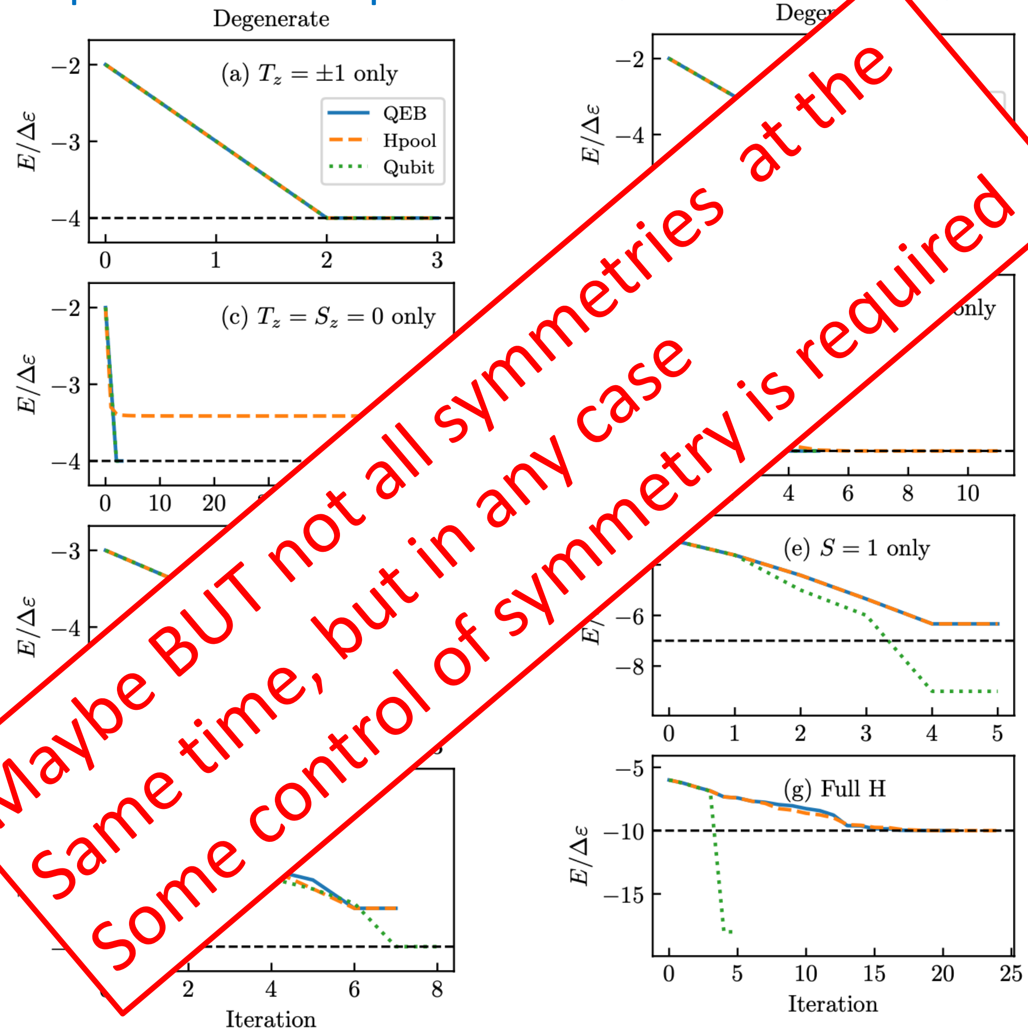
Different operator pool in ADAPT-VQE breaking or not symmetries

	Particle number	Seniority	Parity
H-pool	✓	✓	✓
QEB-pool	✓	×	✓
Qubit-pool	×	×	✓



4 particles on 8 qubits

4 particles on 12 qubits

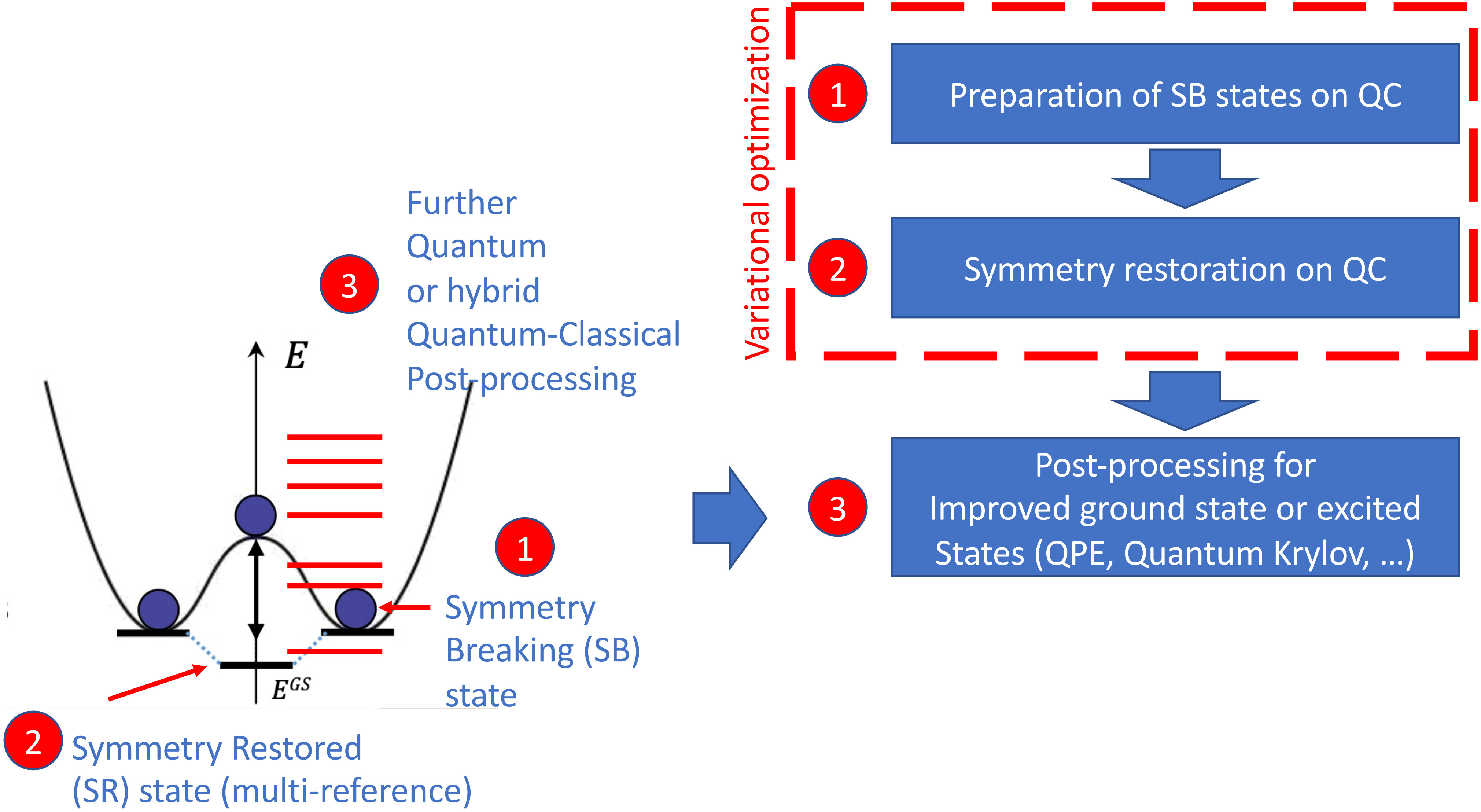


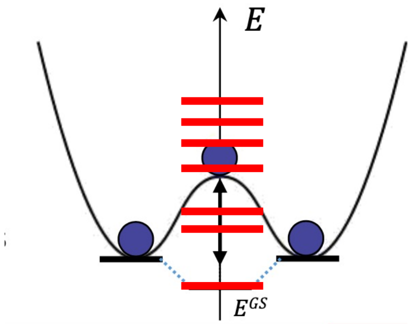
Maybe BUT not all symmetries at the same time, but in any case

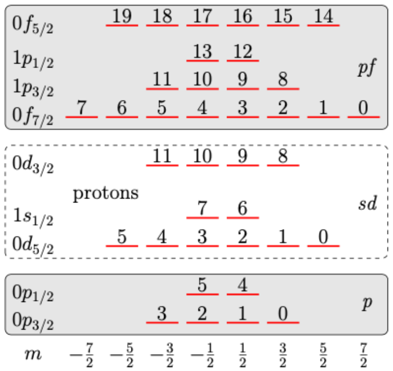
Some control of symmetry is required

J. Zhang, DL, and Y. Beaujeault-Taudière, arXiv:2408.17294

What about excited states?

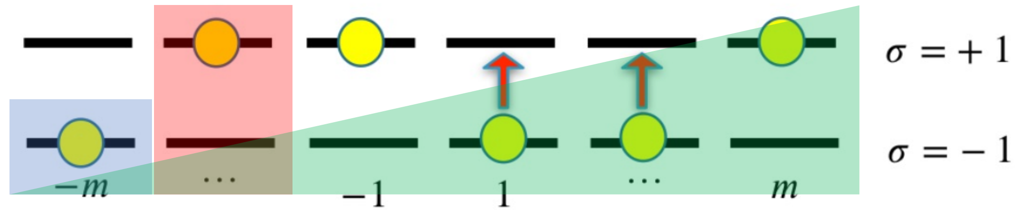






Today's challenges:

- -Identify pilot applications,
- -Reduce the Quantum resources
- -Develop novel quantum algorithms

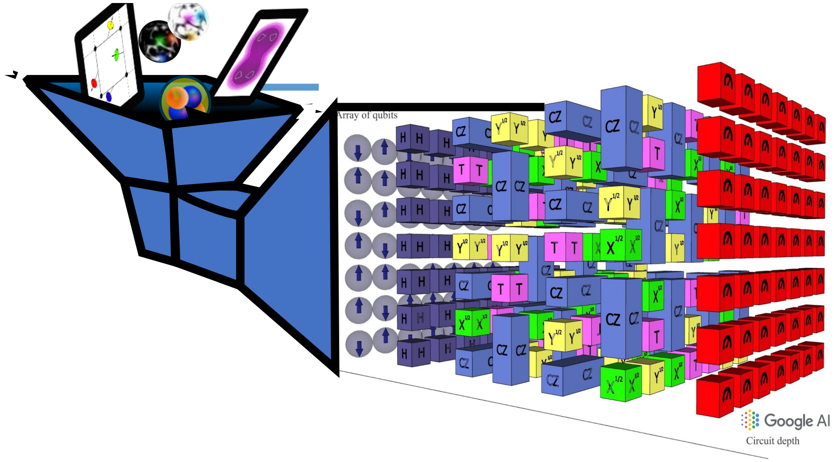


q = Number of qubits

Fermions-to-qubit: Jordan-Wigner	SU(2) encoding	J-scheme (compact) +parity encoding
1 level = 1 qubit	2 levels = 1 qubit	$\|J, M\rangle \to \|[M]\rangle$
$q = 2N$	$q = N$	Use first quantization
		$q = \lceil \log_2 N \rceil$

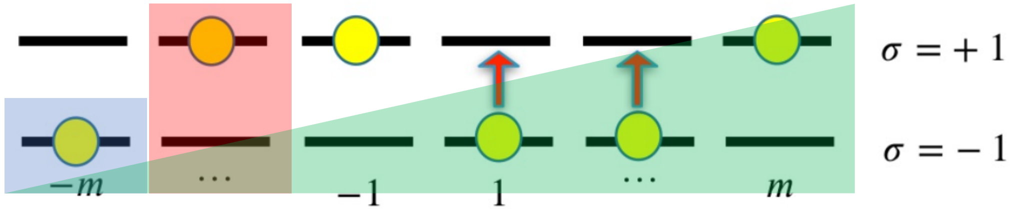
$$\begin{cases} X_\alpha = |1_\alpha\rangle\langle 0_\alpha| + |0_\alpha\rangle\langle 1_\alpha|, \\ Y_\alpha = i|1_\alpha\rangle\langle 0_\alpha| - i|0_\alpha\rangle\langle 1_\alpha|, \\ Z_\alpha = |0_\alpha\rangle\langle 0_\alpha| - |1_\alpha\rangle\langle 1_\alpha| \end{cases}$$

$$H = \frac{\varepsilon}{2}\sum_{\alpha=1}^{N} Z_\alpha + \frac{V}{4}\sum_{\alpha\neq\beta}^{N}(X_\alpha X_\beta - Y_\alpha Y_\beta).$$



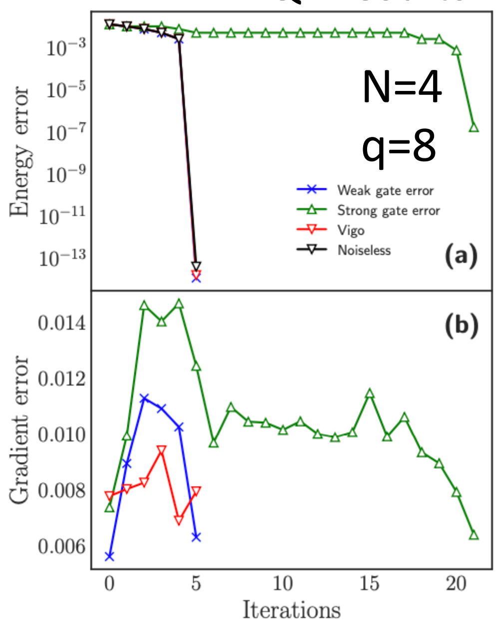
q = Number of qubits

Encoding the Lipkin model on a quantum register



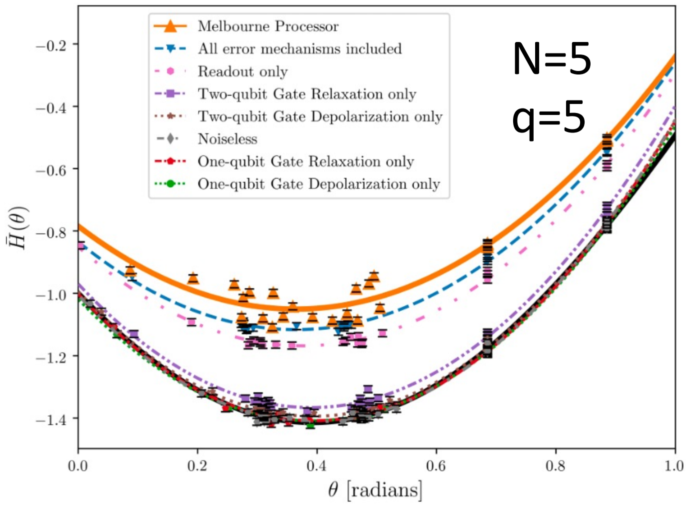
Fermions-to-qubit: Jordan Wigner	SU(2) encoding	J-scheme (compact) +parity encoding

ADAPT-VQE results



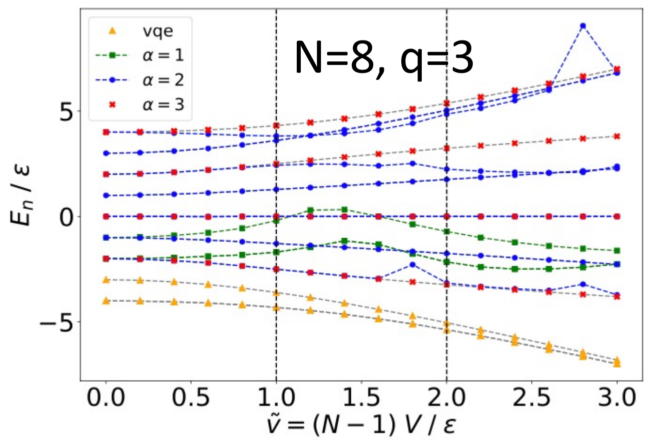
J. Romero et al, PRC 105 (2022)

VQE results



M. Cervia et al, PRC 104 (2021)

QEOM-technique

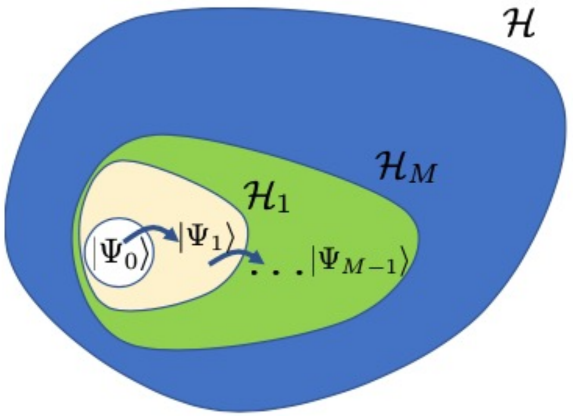


Hlatshwayo et al, PRC 106 (2022), & PRC 109 (2024)

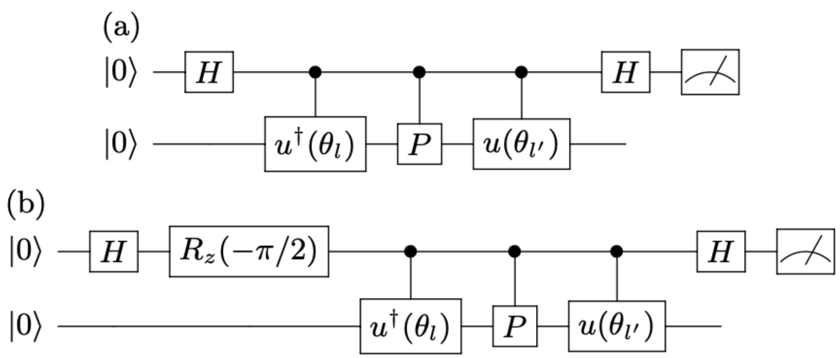
Solving the Lipkin model with using 2 qubits only with hybrid quantum-classical method

Classical post processing

Quantum Subspace expansion



Real/Imaginary parts requires 2 qubits



Quantum Generator Coordinate Method

$$|\Psi\rangle = \int_{\mathbf{q}} f(\mathbf{q})|\Phi(\mathbf{q})\rangle d\mathbf{q}$$

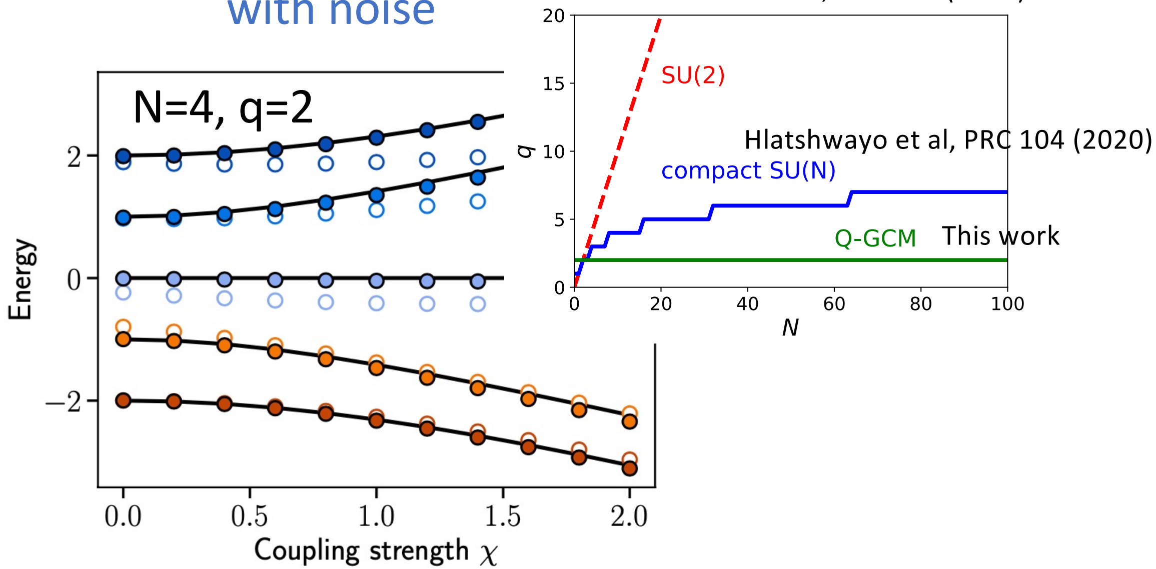
$$\int_{\mathbf{q}'} d\mathbf{q}' \left[\mathcal{H}(\mathbf{q},\mathbf{q}') - E\mathcal{N}(\mathbf{q},\mathbf{q}')\right] f(\mathbf{q}') = 0$$

Lipkin /perm. invariant

$$\langle H\rangle_{ll'} = \frac{\varepsilon N}{2} i_{ll'}^{N-2}\left[i_{ll'}z_{ll'} + \frac{\chi}{2}\left(x_{ll'}^2 - y_{ll'}^2\right)\right]$$

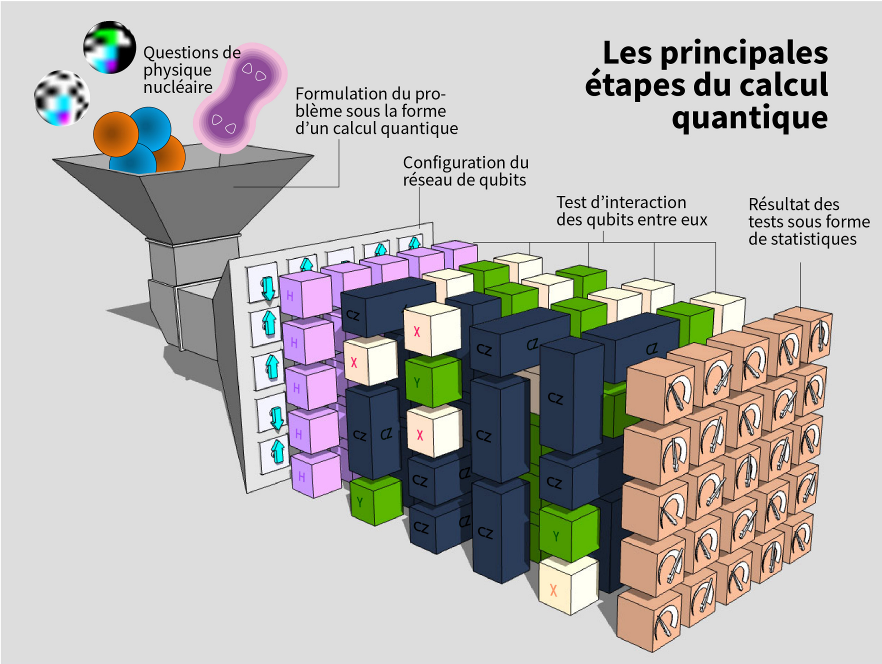
one-body kernels

Illustration of Lipkin model on QC with noise



Y. Beaujeault-Taudière and DL, PRC 109, 024327 (2024)

Some final highlights



- The main goal was to find pilot applications for Quantum Computing where Qc can be disruptive compared to CC
- We first focused on the problem of Many-body / symmetries (that could be useful beyond many-body problems) - and developed various Techniques (QPE, Oracle, Shadow, Hilbert Space expansion, … mostly post NISQ techniques)
- We are now gaining expertise on algorithmic, noise, …
- Our current interest are: variational ansatz and expressivity, equilibrium and non-equilibrium properties, entanglement growth, …

We have regular opportunities of PhD & Postdoc

Thank you!